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ABSTRACT

Fundamental concepts of motion are presented in this first unit of the Project Physics textbook. Descriptions of motion are made in connection with speeds, accelerations, and their graphical representation. Free-fall bodies are analyzed by using Aristotle's theory and Galileo's work. Dynamics aspects are discussed with a background of mass, force, vectors, laws of motion, equilibrium forces, weight, gravitation, and nature's basic forces. Further information is given in terms of the earth-moon trip, path of a projection, Galilean relativity, circular motion, centripetal acceleration, motion of earth satellites, and simple harmonic motion. Historical developments are stressed in explanation of concepts. A description of Fermi's work and tables of man's place in time and space are included in the prologue. Questions are given at the end of each section as well as at the end of the whole text. Besides illustrations for explanation use, brief answers to the questions are provided. The work of Harvard Project Physics has been financially supported by: the Carnegie Corporation of New York, the Ford Foundation, the National Science Foundation, the Alfred P. Sloan Foundation, the United States Office of Education, and Harvard University. (CC)

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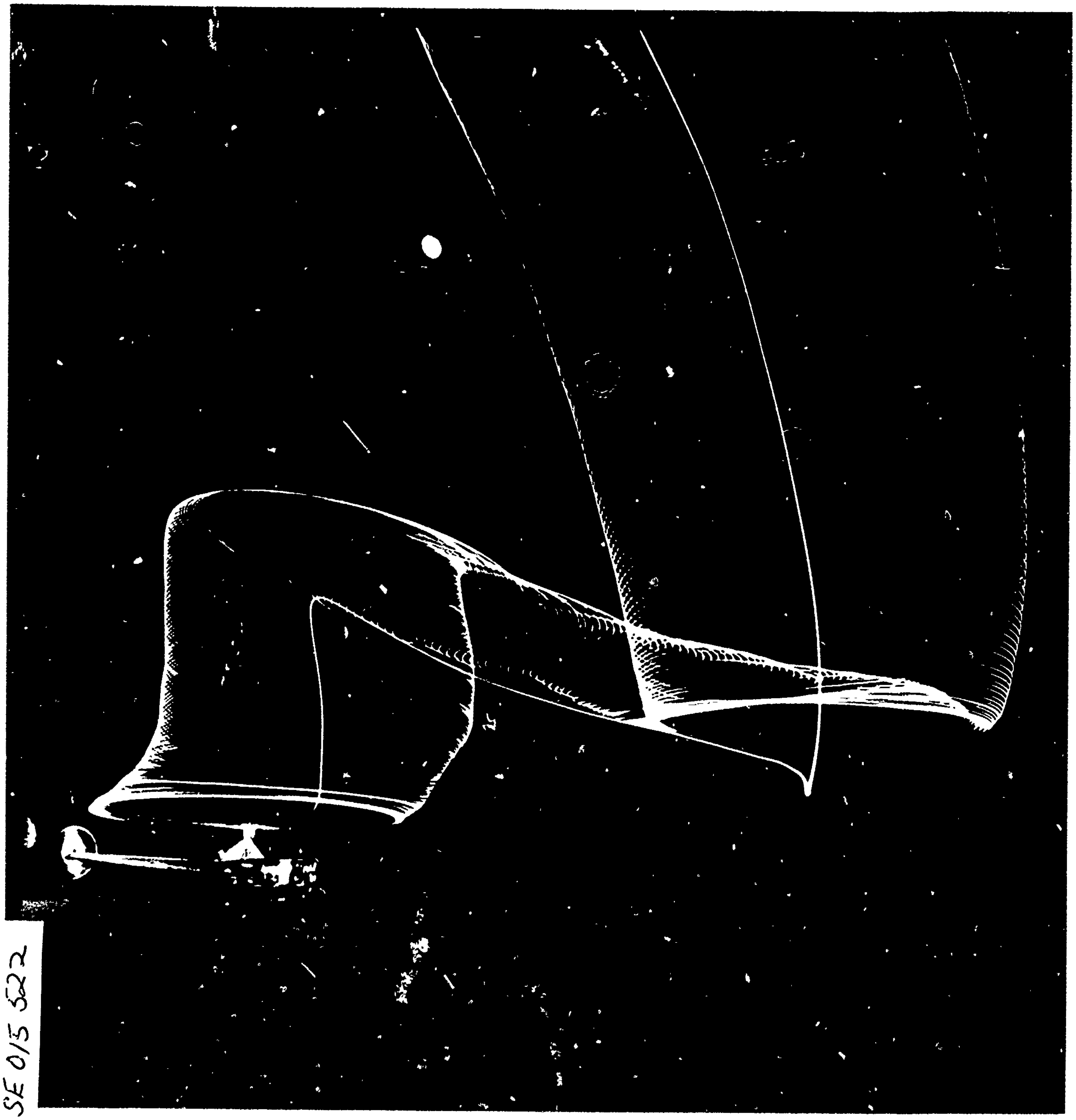
Project Physics **Text** **1**

An Introduction to Physics

Concepts of Motion



SE 015 522

Project Physics **Text**

An Introduction to Physics 1 Concepts of Motion

Authorized Interim Version 1968-69

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The Project Physics course has been developed through the creative assistance of many colleagues. The following is a partial list of those contributors (the affiliations indicated are those just prior to or during their association with the Project).

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Welcome to the study of physics. This volume, more of a student's guide than a text of the usual kind, is part of a whole group of materials that includes a student handbook, laboratory equipment, films, programmed instruction, readers, transparencies, and so forth. Havrard Project Physics has designed the materials to work together. They have all been tested in classes that supplied results to the Project for use in revisions of earlier versions.

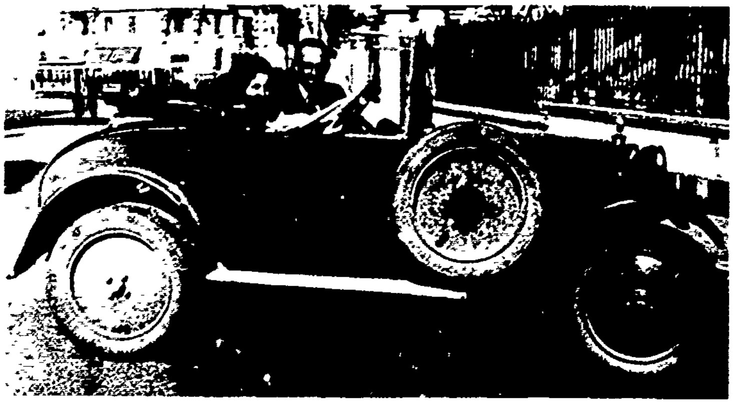
The Project Physics course is the work of about 200 scientists, scholars, and teachers from all parts of the country, responding to a call by the National Science Foundation in 1963 to prepare a new introductory physics course for nationwide use. Harvard Project Physics was established in 1964, on the basis of a two-year feasibility study supported by the Carnegie Corporation. On the previous pages are the names of our colleagues who helped during the last six years in what became an extensive national curriculum development program. Some of them worked on a full-time basis for several years; others were part-time or occasional consultants, contributing to some aspect of the whole course; but all were valued and dedicated collaborators who richly earned the gratitude of everyone who cares about science and the improvement of science teaching.

Harvard Project Physics has received financial support from the Carnegie Corporation of New York, the Ford Foundation, the National Science Foundation, the Alfred P. Sloan Foundation, the United States Office of Education and Harvard University. In addition, the Project has had the essential support of several hundred participating schools throughout the United States and Canada, who used and tested the course as it went through several successive annual revisions.

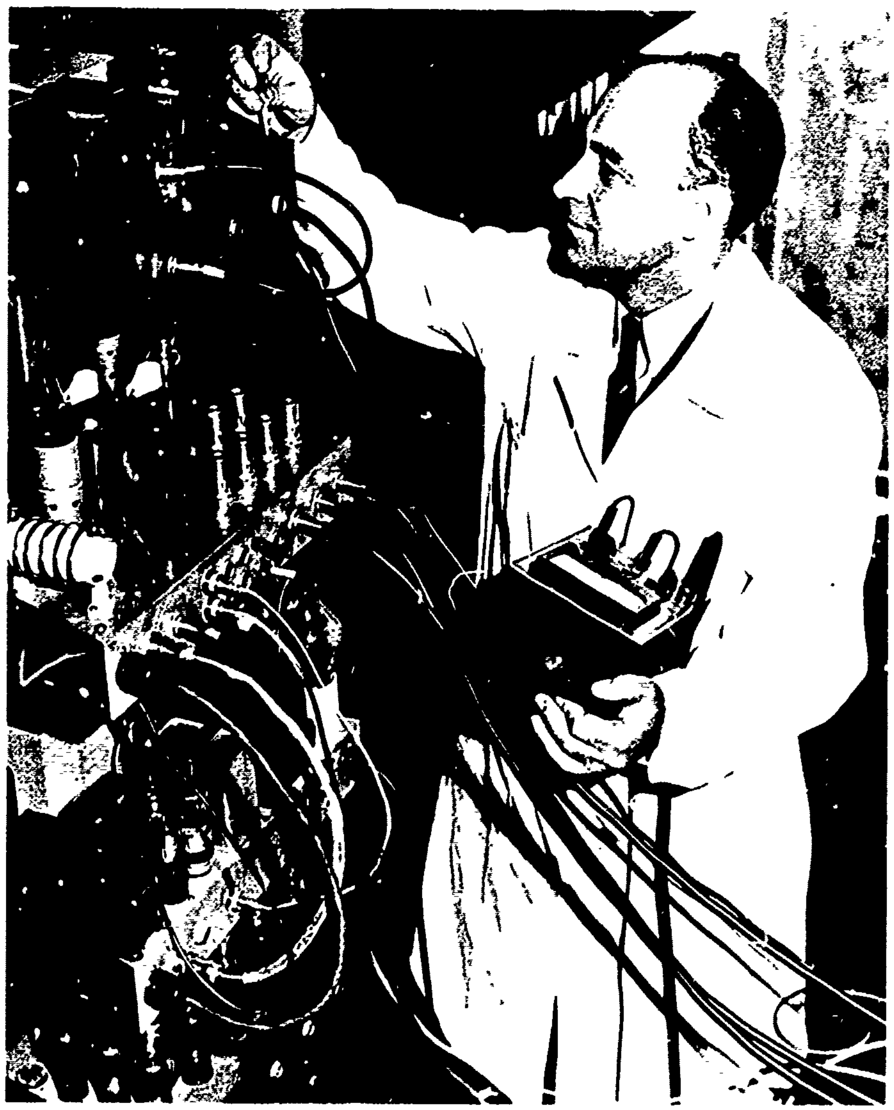
The last and largest cycle of testing of all materials is now completed; the final version of the Project Physics course will be published in 1970 by Holt, Rinehart and Winston, Inc., and will incorporate the final revisions and improvements as necessary. To this end we invite our students and instructors to write to us if in practice they too discern ways of improving the course materials.

The Directors
Harvard Project Physics

An Introduction to Physics 1 Concepts of Motion



The physicist, Enrico Fermi (1901-1954), at different stages of his career in Italy and America. The photographs were kindly supplied by Mrs. Laura Fermi, shown also in the top photograph.



Prologue It is January of 1934, a dreary month in the city of Paris, and a husband and wife are bombarding a bit of aluminum with what are called alpha particles. Does this seem like a momentous event? Certainly not when stated so baldly. But let us look at it more closely, for it is momentous indeed.

Never mind the technical terms. They will not get in the way of the story. It begins as something of a family affair. The husband and wife, French physicists, were Frédéric Joliot and Irene Curie, and the alpha particles they used in their experiment came shooting out of a radioactive metal, polonium, discovered 36 years before by none other than Irene's illustrious parents, Pierre and Marie Curie, who also discovered radium. What Frederic and Irene found was this: when bombarded by alpha particles, the commonplace bit of aluminum became radioactive.

Nothing like this had ever been observed before: a familiar, everyday substance becoming radioactive. The news was exciting to scientists—though it made few, if any newspaper headlines. The news traveled rapidly: by cablegram and letter. In Rome, Enrico Fermi, a young physicist on the staff at the University of Rome, became intrigued by the possibility of repeating the experiment of Frédéric and Irene—repeating it with one significant alteration. The story is told in the book Atoms in the Family by Enrico Fermi's wife, Laura. She writes:

> ...he decided he would try to produce artificial radioactivity with neutrons [instead of alpha particles]. Having no electric charge, neutrons are neither attracted by electrons nor repelled by nuclei; their path inside matter is much longer than that of alpha particles; their speed and energy remain higher; their chances of hitting a nucleus with full impact are much greater. Against these unquestionable advantages, neutrons present a decidedly strong drawback. Unlike alpha particles, they are not emitted spontaneously by radioactive substances, but they are produced by bombarding certain elements with alpha particles, a process yielding approximately one neutron for every hundred thousand alpha particles. This very low yield made the use of neutrons appear questionable.

All quotations in the Prologue are from Laura Fermi, Atoms in the Family: My Life With Enrico Fermi, University of Chicago Press, 1954 (available as a paperback book in the Phoenix Books series). Fermi was one of the major physicists of this century.

Only through actual experiment could one tell whether or not neutrons were good projectiles for triggering artificial radioactivity of the target nuclei. Therefore, Fermi, at the age of 33 and already an outstanding theoretical physicist, decided to design some experiments that could settle the issue. His first task was to obtain suitable instruments for detecting the particles emitted by radioactive materials. By far the best such instruments were what are called Geiger counters, but in 1934, Gei⁻er counters were still relatively new and not readily available. Therefore, Fermi constructed his own.

The counters were soon finished, and tests showed that they could detect the radiation from radioactive materials. But Fermi also needed a source of neutrons. This he made by enclosing beryllium powder and the radioactive gas radon in a glass tube; the alpha particles from radon, on striking the beryllium, caused it to emit neutrons.

> Now Enrico was ready for the first experiments. Being a man of method, he did not start by bombarding substances at random, but proceeded in order, starting from the lightest element, hydrogen, and following the periodic table of elements. Hydrogen gave no results: when he bombarded water with neutrons, nothing happened. He tried lithium next, but again without luck. He went on to beryllium, then to boron, to carbon, to nitrogen. None were activated. Enrico wavered, discouraged, and was on the point of giving up his researches, but his stubbornness made him refuse to yield. He would try one more element. That oxygen would not become radioactive he knew already, for his first bombardment had been on water. So he irradiated flourine. Hurrah! He was rewarded. Fluorine was strongly activated, and so were other elements that came after fluorine in the periodic table.
>
> This field of investigation appeared so fruitful that Enrico not only enlisted the help of Emilio Segré and of Edoardo Amaldi but felt justified in sending a cable to Rasetti [a colleague who ha gone to Morocco], to inform him of the experiments .d to advise him to come back at once. A short later a chemist, Oscar D'Agostino, joined the group, and systematic investigation was carried on at a fast pace.

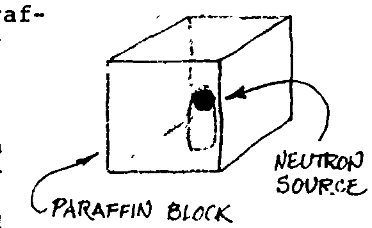
With the help of his colleagues, Fermi's work at the laboratory was pursued with high spirit, as Laura Fermi's account shows:

> ...Irradiated substances were tested for radioactivity with Geiger counters. The radiation emitted by the neutron source would have disturbed the measurements had it reached the counters. Therefore, the room where substances were irradiated and the room with the counters were at the two ends of a long corridor.
>
> Sometimes the radioactivity produced in an element was of short duration, and after less than a minute it could no longer be detected. Then haste was essential, and the time to cover the length of the corridor had to be reduced by swift running. Amaldi and Fermi prided themselves on being the fastest runners, and theirs was the task of speeding short-lived substances from one end of the corridor to the other. They always raced, and Enrico claims that he could run faster than Edoardo....

Again, follow the story to get a feeling for the atmosphere of important experiments—don't worry about details now.

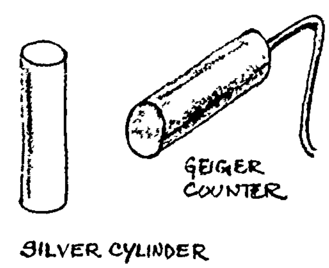
And then, on the morning of October 22, 1934, a fateful discovery was made. Two of Fermi's co-workers were irradiating a hollow cylinder of silver with neutrons from a source placed at the center of the cylinder, to make it artificially radioactive. They found that the amount of radioactivity induced in the silver depended on other objects in the room!

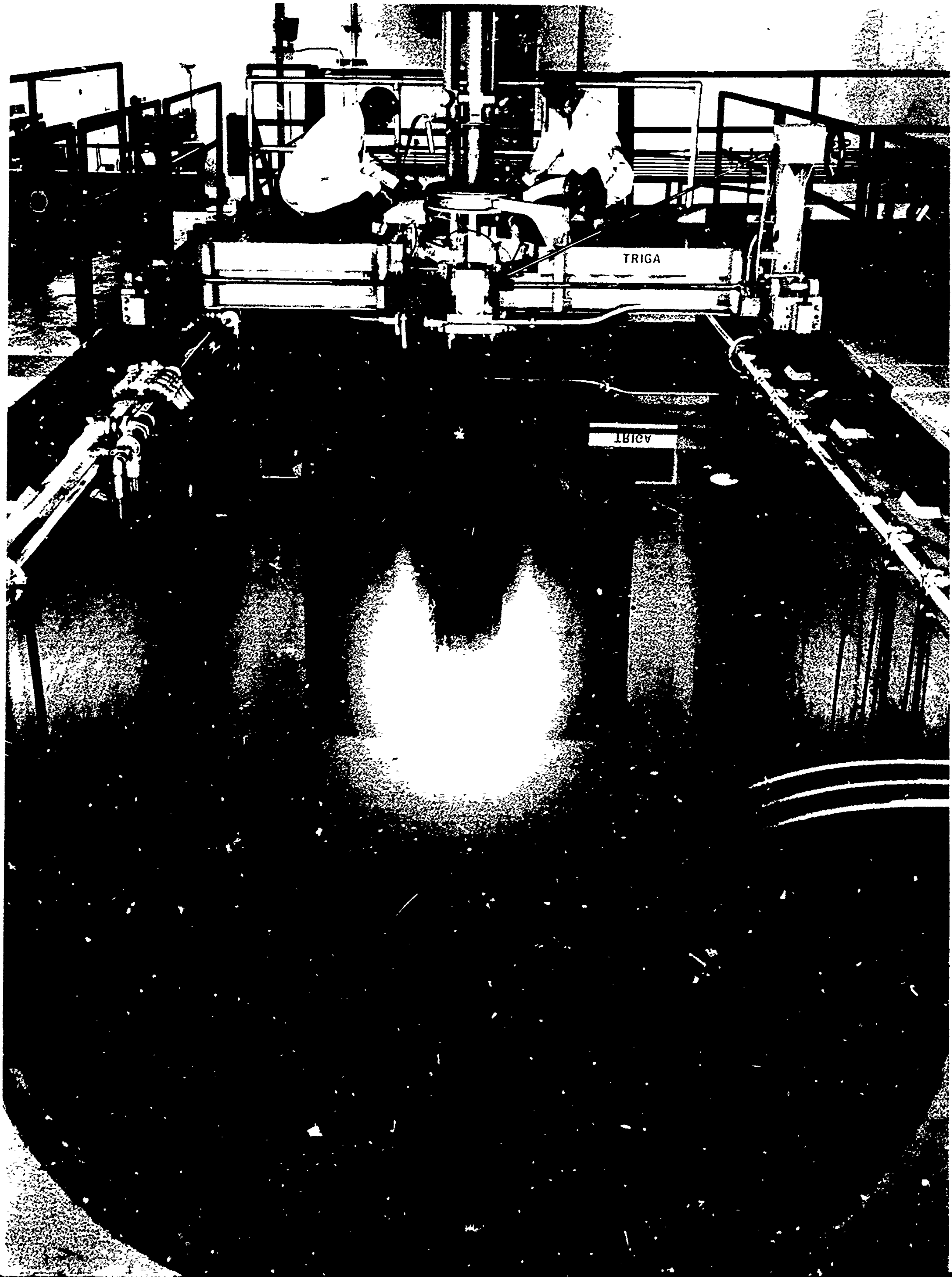
> ...The objects around the cylinder seemed to influence its activity. If the cylinder had been on a wooden table while being irradiated, its activity was greater than if it had been on a piece of metal. By now the whole group's interest had been aroused, and everybody was participating in the work. They placed the neutron source outside the cylinder and interposed objects between them. A plate of lead made the activity increase slightly. Lead is a heavy substance. "Let's try a light one next," Fermi said, "for instance, paraffin." [The most plentiful element in paraffin is hydrogen.] The experiment with paraffin was performed on the morning of October 22.
>
> They took a big block of paraffin, dug a cavity in it, put the neutron source inside the cavity, irradiated the silver cylinder, and brought it to a Geiger counter to measure its activity. The counter clicked madly. The halls of the physics building resounded with loud exclamations: "Fantastic! Incredible! Black Magic!" Paraffin increased the artificially induced radioactivity of silver up to one hundred times.



By the time Fermi came back from lunch, he had already formulated a theory to account for the strange action of paraffin.

> Paraffin contains a great deal of hydrogen. Hydrogen nuclei are protons, particles having the same mass as neutrons. When the source is inclosed in a paraffin block, the neutrons hit the protons in the paraffin before reaching the silver nuclei. In the collision with a proton, a neutron loses part of its energy, in the same manner as a billiard ball is slowed down when it hits a ball of its same size [whereas it loses little speed if it is reflected off a much heavier ball, or a solid wall]. Before emerging from the paraffin, a neutron will have collided with many protons in succession, and its velocity will be greatly reduced. This <u>slow</u> neutron will have a much better chance of being captured by a silver nucleus than a fast one, much as a slow golf ball has a better chance of making a hole than one which zooms fast and may bypass it.
>
> If Enrico's explanations were correct, any other substance containing a large proportion of hydrogen should have the same effect as paraffin. "Let's try and see what a considerable quantity of water does to the silver activity," Enrico said on the same afternoon.
>
> There was no better place to find a "considerable quantity of water" than the goldfish fountain...in the garden behind the laboratory....




TRIGA

In that fountain the physicists had sailed certain small toy boats that had suddenly invaded the Italian market. Each little craft bore a tiny candle on its deck. When the candles were lighted, the boats sped and puffed on the water like real motor-boats. They were delightful. And the young men, who had never been able to resist the charm of a new toy, had spent much time watching them run in the fountain.

It was natural that, when in need of a considerable amount of water, Fermi and his friends should think of that fountain. On that afternoon of Octobr they rushed their source of neutrons and thei er cylinder to that fountain, and they placed both under water. The goldfish, I am sure, retained their calm and dignity, despite the neutron shower, more than did the crowd outside. The men's excitement was fed on the results of this experiment. It confirmed Fermi's theory. Water also increased the artificial radioactivity of silver by many times.

The same process by which neutrons were slowed down in the fountain is used in today's large nuclear reactors. An example is the "pool" research reactor pictured on the opposite page.

This discovery—that slowed-down neutrons can produce much stronger effects in the transmutation of certain atoms than fast neutrons—turned out to be a crucial step toward further discoveries that, years later, led Fermi and others to the extraction of atomic energy from uranium.

The reason for presenting a description of Fermi's discovery of slow neutrons here was <u>not</u> to instruct you on the details of the nucleus. It was, instead, to present a quick, almost impressionistic, view of scientists in action. No other discovery in science was made or will be made in just the way Fermi and his colleagues made this one. Nevertheless, the episode does illustrate some of the characteristics—and some of the drama—of modern science.

Like religion, science probably began as a sense of awe and wonder. In its highest form its motive power has been sheer curiosity—the urge to explore and to know. This urge is within us all. It is vividly seen in the intense absorption of a child examining a strange sea shell tossed up from the ocean or a piece of metal found in the gutter. Who among us has resisted the temptation to explore the slippery properties of the mud in a rain puddle? Alas, everyday cares and the problems of growing up overtake us all too soon, and many of us lose our early sense of curiosity or channel it into more practical paths. Fortunately, a few preserve their childlike, wide-eyed wonderment and it is among such people that one often finds the great scientists and poets.

Science gives us no final answers. But it has come upon wondrous things, and some of them may renew our childhood delight in the miracle that is within us and around us. Take, for example, so basic a thing as size...or time.

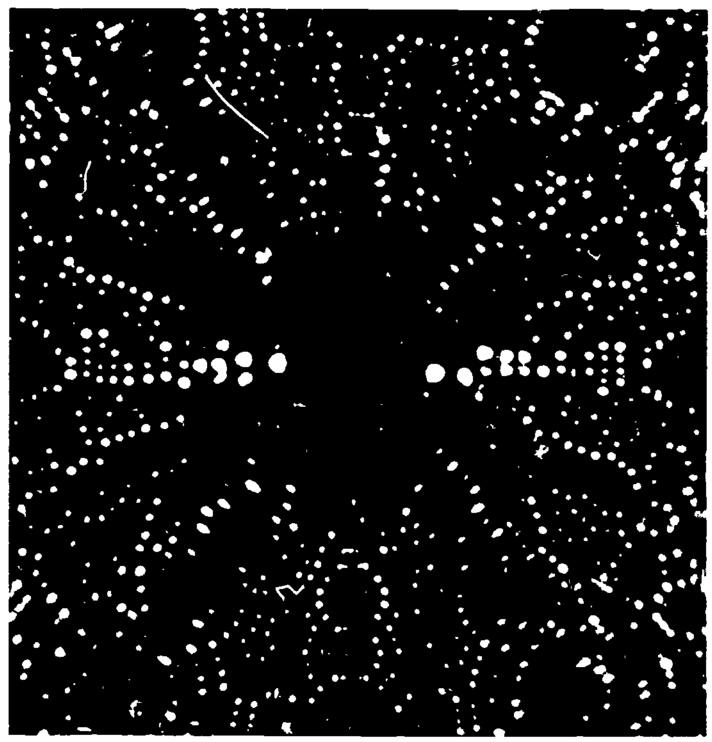
Our Place in Space

	(orders of magnitude)	
Size of universe	10^{26}	m
Distance to nearest galaxy	10^{22}	m
Distance to nearest star	10^{17}	m
Distance to the sun	10^{11}	m
Diameter of earth	10^{7}	m
One mile	10^{3}	m
Human height	10^{0}	m
Postage stamp	10^{-2}	m
Paper thickness	10^{-4}	m
Atomic diameter	10^{-10}	m
Nuclear diameter	10^{-14}	m

A globular star cluster

The estimated size of the universe now extends to more than a million, million, millio-, million times a man's height.

Atomic sites in tungsten



The smallest known constituent units of the universe are less in size than 1,000 million millionths of a man's height.

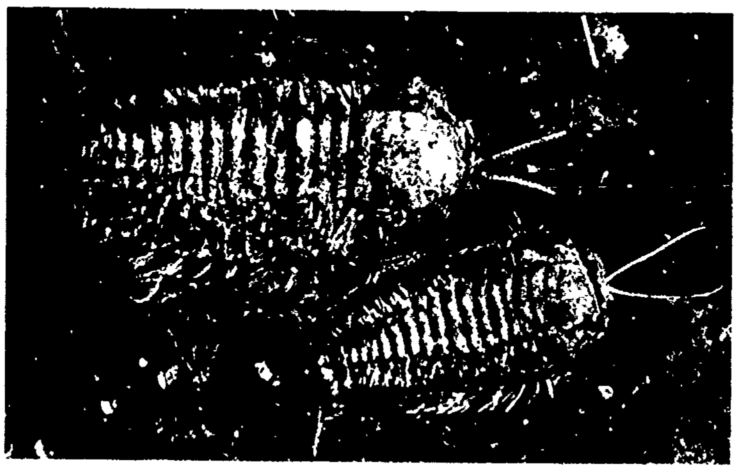
man's height x 1,000,000,000,000,000,000,000,000

man's height x 0.000,000,000,000,001

Our Place in Time

	(orders of magnitude)
Expected life of sun	10^{18} sec
Age of earth	10^{16} sec
Human life span	10^{9} sec
One day	10^{5} sec
Light from sun to earth	10^{3} sec
Time between heartbeats	10^{0} sec
One beat of fly's wings	10^{-3} sec
Average lifetime of a muon	10^{-6} sec
Average lifetime of pi meson	10^{-15} sec
Shortest-lived particles	10^{-23} sec

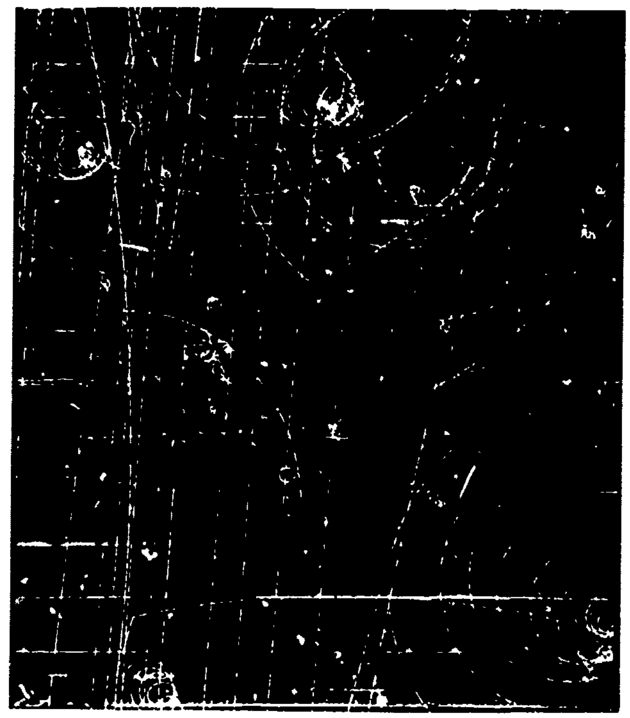
Fossilized trilobites



The history of the universe has been traced back as far into the past as a hundred million times the length of a man's life.

man's life x 100,000,000

Particle tracks in a bubble chamber

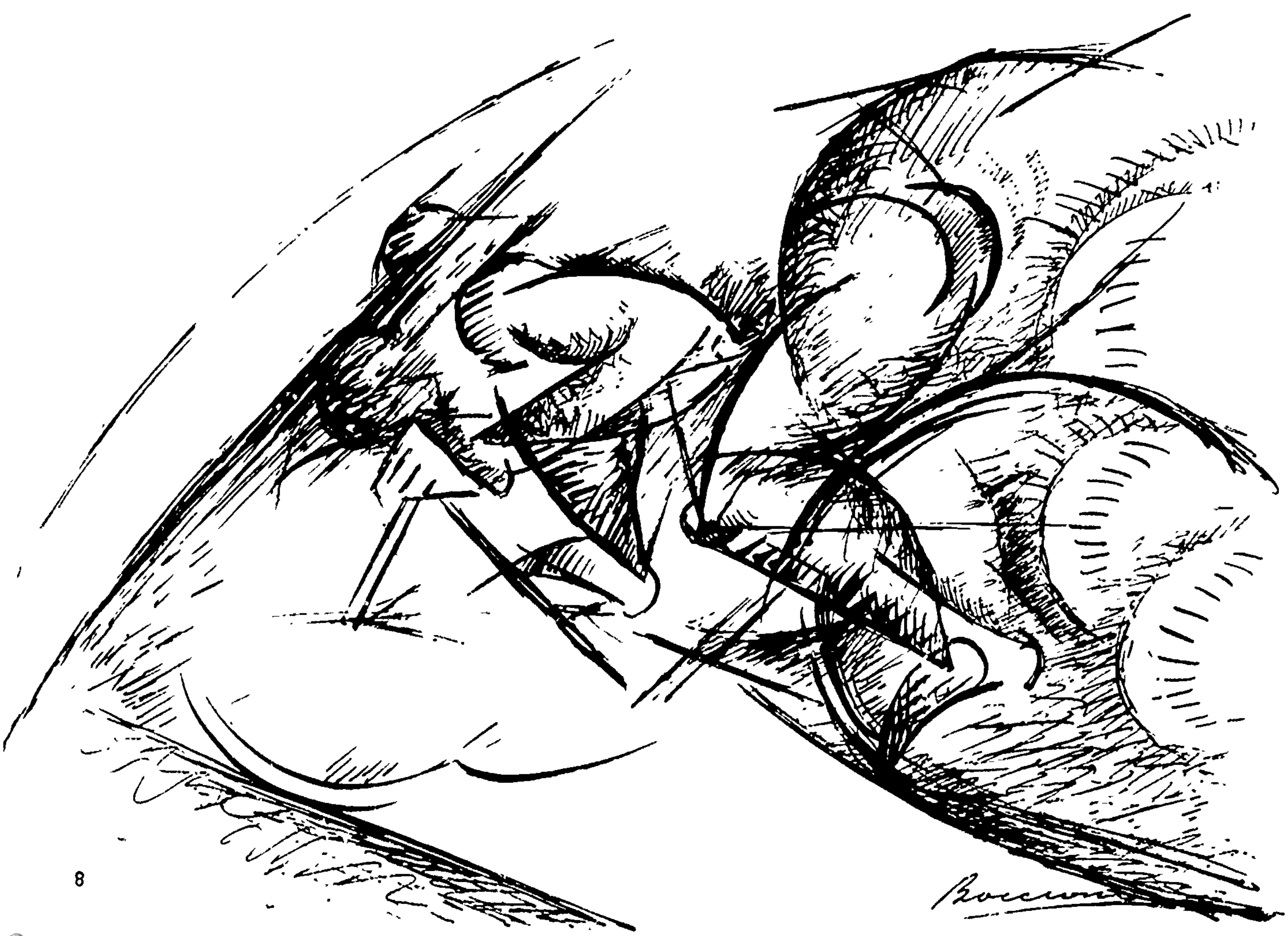


Events have been recorded that last only a millionth of a millionth of a millionth of a millionth of a man's heartbeat.

man's heartbeat x 0.000,000,000,000,000,000,000,001

It is hard to resist the temptation to say more about these intriguing extremes; however, this is not where physics started. Physics started with the man-sized world—the world of horse-drawn chariots, of falling rain, and of flying arrows. It is with this man-sized world that we shall begin.

Chapter 1 The Language of Motion



There is a very old maxim: "To be ignorant of motion is to be ignorant of Nature."

1.1 <u>The motion of things</u>. Man crawls, walks, runs, jumps, dances. To move himself faster, farther, higher, deeper, he invents things like sleds, bicycles, submarines, rocket ships. As human beings we are caught up in motion and fascinated by it. Perhaps this is why so many artists try to portray movement. It is one reason why scientists investigate motion. The world is filled with things in motion: things as small as dust, as large as stars, and as common as ourselves; motion fast and slow, motion smooth, rhythmic, and erratic. We cannot investigate all of these at once. So from this swirling, whirling world of ours let us choose just one moving object for attention, something interesting and typical, and, above all, something manageable.

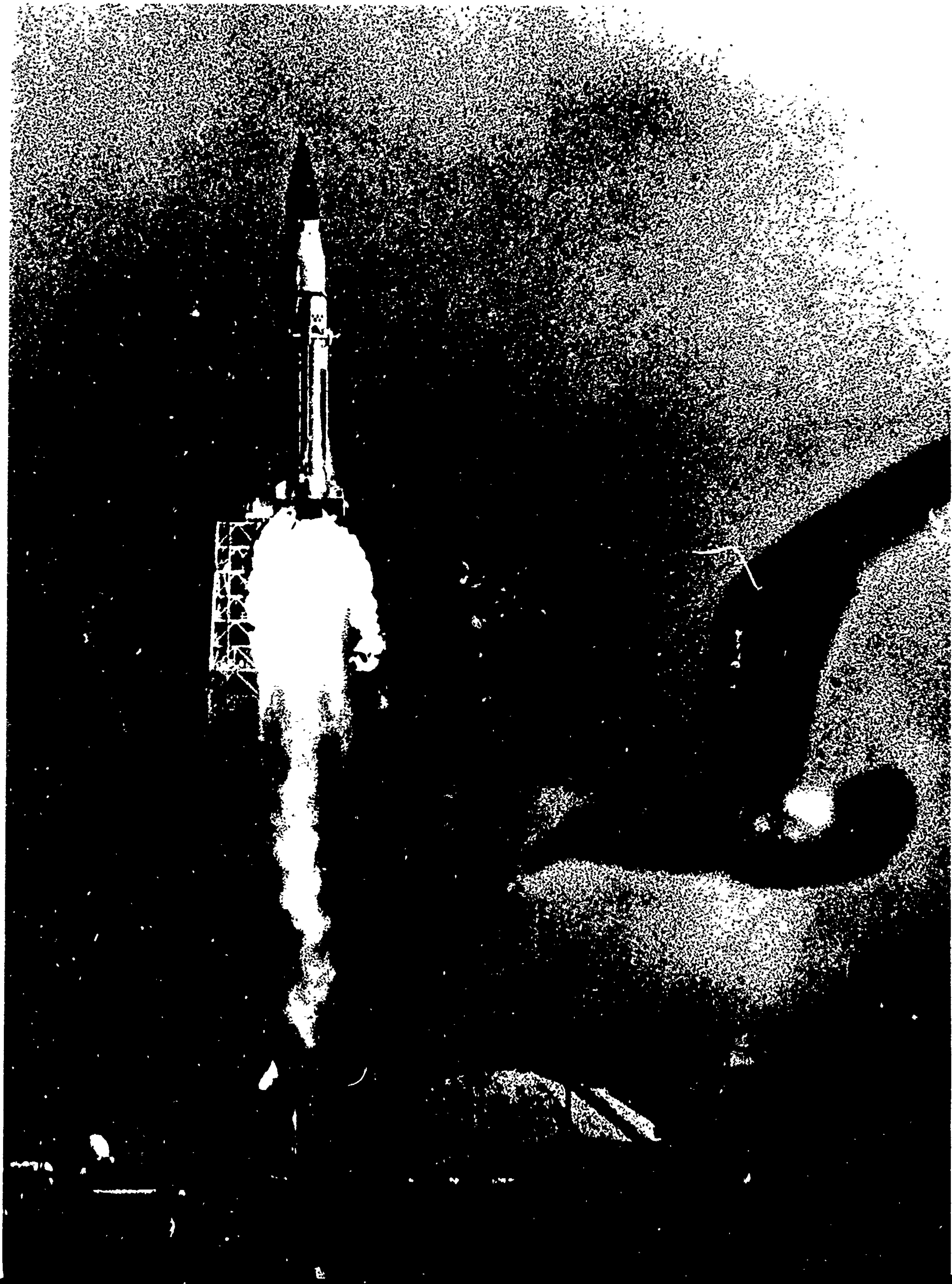
But where shall we start? We might start our investigation by looking at a modern machine—the Saturn rocket, say, or a supercharged dragster, or an automatic washing machine. But as you know, things such as these, though made and controlled by man, move in very complicated ways. We really ought to start with something easier. Then how about the bird in flight? Or a leaf falling from a tree?

Surely in all of nature there is no motion more ordinary than that of a leaf fluttering down from a branch. Can we describe how it falls or explain why it falls? As we think about it we quickly realize that, while the motion may be natural, it is very complicated: the leaf twists, turns, sails to the right and left, back and forth, as it floats down. Even a motion as ordinary as this may turn out, on closer examination, to be more complicated than that of machines. Although we might describe it in detail, what would we gain? No two leaves fall in quite the same way; therefore, each leaf would require its own detailed description. Indeed, this individuality is typical of many naturally occurring events on earth!

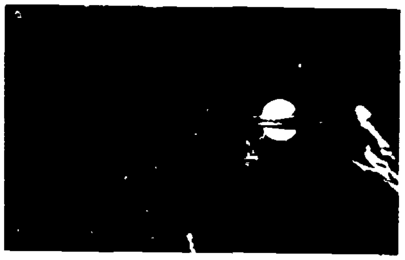
And so we are faced with a real dilemma. We want to describe motion, but the motions that excite and interest us appear to be hopelessly complex. What shall we do? We shall find a very simple motion and attempt to describe it. Those of us who have learned to play a musical instrument will appreciate the wisdom of starting with simple tasks. If our music teacher confronted us in lesson number one with a Beethoven piano sonata, we would in all probability have quickly forgone music in favor of a less taxing activity.

The place to start is in the laboratory, because there we can find the simple ingredients that make up complex motions.

Study for "Dynamism of a Cyclist" (1913) by Umberto Boccioni. Courtesy Yale University Art Gallery.

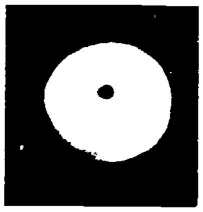


1.2 <u>A motion experiment that does not quite work</u>. A billiard ball hit squarely in the center speeds across the table in a straight line. Unfortunately, physics laboratories are not usually equipped with billiard tables. But never mind. Even better for our purposes is a disc of what is called dry ice (really frozen carbon dioxide) moving on the floor. The dry ice disc was placed on the floor and given a gentle push. It floated slowly across the floor in front of the camera. While the disc was moving, the shutter of the camera was kept open. The resultant time exposure shows the path taken by the dry ice disc.

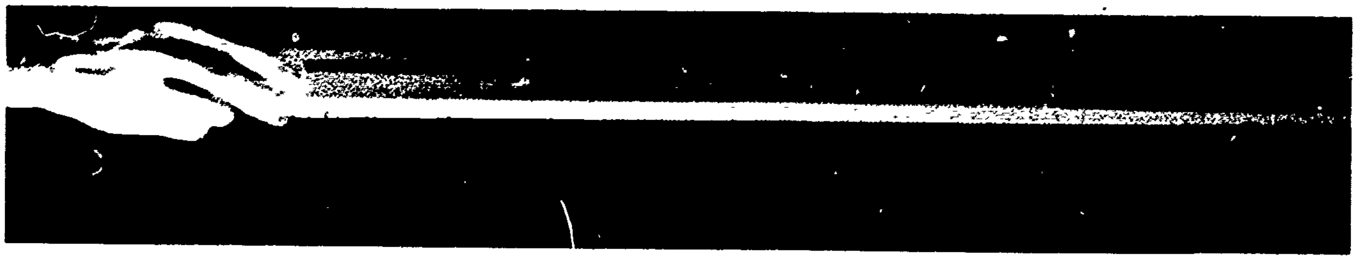


See Study Guide 1.1 (page 32)

Laboratory setup



Close-up of a dry ice disc



Time exposure of the disc in motion

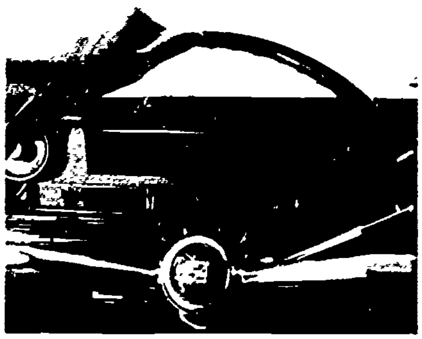
What can we learn about the disc's motion by examining the photographic record? Was the path a straight line? Did the disc slow down?

The question of path is easy enough to answer: as nearly as we can judge by placing a ruler on the photograph, the disc moved in a straight line. But did it slow down? From the photograph we cannot tell. Let us improve our experiment. Before we do so, however, we ought to be clear on just how we might expect to measure speed.

Why not use something like an automobile speedometer? All of us know how to read that most popular of all meters even though we may not have a clear notion of how it works. A speedometer tells us directly the speed at which the car is moving at any time. Very convenient. Furthermore, such

We are assuming here that you already know what speed <u>is</u>, namely how fast an object moves from one place to another. A more formal way to say the same thing is: <u>Speed is the time rate of change of position</u>.

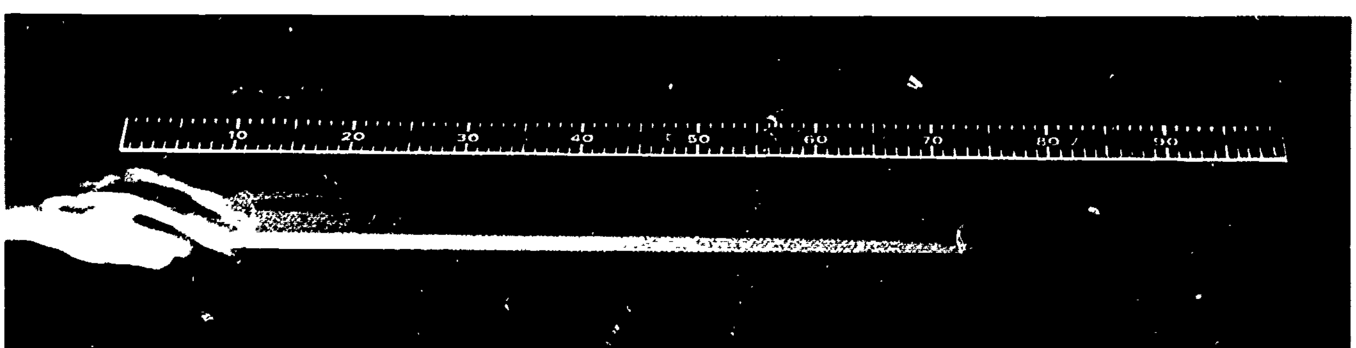
readings are independent of the path of our car. A given speedometer reading specifies the same speed, whether the car is moving uphill or down, or is traveling along a straight road or a curved one.



But, alas, there is at least one practical trouble with having to rely on a speedometer to measure speed: it is not easy to put a speedometer on a disc of dry ice, or on a bullet, or on many other objects whose speed you may wish to measure. However, the speedometer provides us with a good clue. Remember how we express speedometer readings? We say our car is moving 60 miles per hour. Translation: at the instant the reading was taken, the car was traveling fast enough to move a distance of 60 miles in a time interval of 1.0 hour, or 120 miles in 2.0 hours, or 6.0 miles in 1/10 hour—or any distance and corresponding time interval for which the ratio of distance to time is 60 miles per hour. To find speed we measure a distance moved, measure the time it took to move that distance, and then divide distance by time.

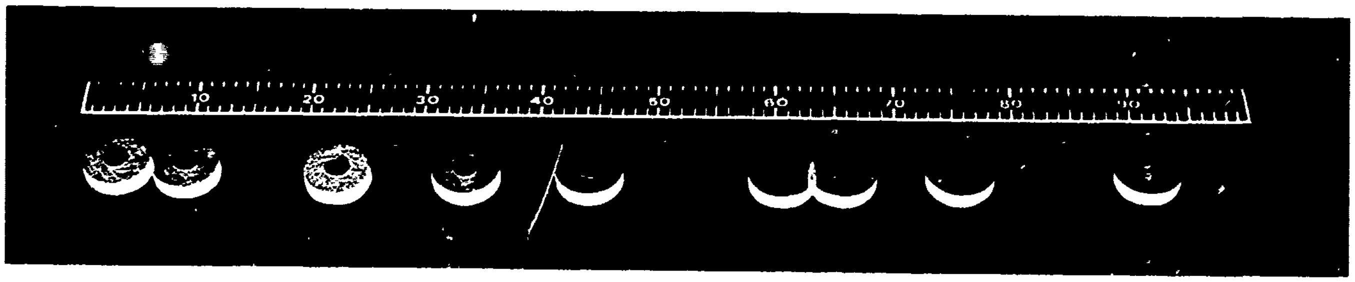
With this reminder of how to measure speed (without a speedometer), we can now return to the experiment with the dry ice disc. Our task now is to redesign the experiment so that we can find the speed of the disc as it moves along its straight-line path.

1.3 A better motion experiment. To find speed we need to be able to measure both distance and time. So let's repeat the experiment with the dry ice disc after first placing a meter stick on the table parallel to the expected path of the disc. This is the photograph we obtain:



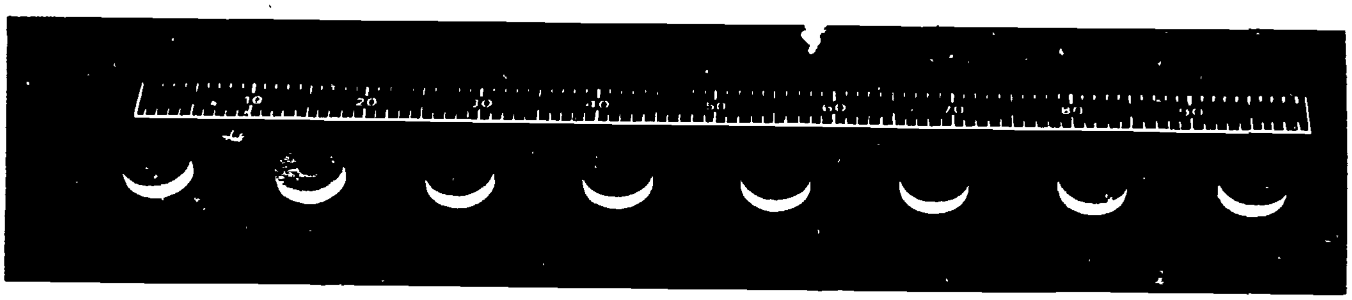
Now the total distance traveled by the disc during the exposure can be measured. However, we still need to measure the time required for the disc to move through a particular distance. However, even if we could measure both the distance and the time we would still have too little information about the motion of the disc. Specifically, to find out whether or not the disc is slowing down—and, if so, by how much—we must be able to find its speed at different places. To do this, we must somehow obtain distance and time information for different places along the path. Knowing only the total distance and total time is not enough.

So let's try another modification. Instead of leaving the camera shutter open, we can open and close it rapidly. The result will be the multiple-exposure photograph shown below.



Although we now have a variety of distances to measure, we still need to know the elapsed time between each exposure. With such information we could analyze the motion in detail, obtaining the distance-to-time ratio (speed) for various segments of the trip. One final change in the apparatus makes this possible.

The camera shutter is again kept open and everything else is the same as before, except that the only source of light in a darkened room comes from a stroboscopic lamp. This lamp flashes very brightly at a constant rate. Since each pulse or flash of light lasts for only about one-millionth of a second, we get a series of separate sharp exposures rather than a continuous, blurred one. The photograph below was made using such a stroboscopic lamp flashing 10 times a second.



Now we're finally getting somewhere. Our experiment enables us to record accurately many positions of a moving object. The meter stick measures the distance the disc moved between successive light flashes. The time elapsed between images is determined by the stroboscopic lamp flashes.

How much did the disc slow down? We can find out by determining its speed at the two ends of its path. The front edge of the first clear image of the disc at the left is 6.0 cm from the zero mark on the meter stick. The front edge of the second image from the left is at the position 19.0 cm. The distance traveled during that interval of time is the difference between those two positions, or 13.0 cm. The corresponding time interval is 0.10 sec. Therefore, the speed at the start must have been 13.0 cm/0.10 sec, or 130 cm/sec.

Turning now to the two images farthest to the right in the photograph, we find that the distance traveled during 0.1 sec was 13.0 cm. Thus, the speed at the end was 13.0 cm/0.1 sec, or 130 cm/sec.

The disc did not slow down at all! The disc's speed was 130 cm/sec at the beginning of the path—and 130 cm/sec at the end of the path. As nearly as we can tell from this experiment, the speed was constant.

That result is hard to believe. Perhaps you are thinking that the disc might have changed speed several times as it moved from left to right but just happened to have identical speeds over the two intervals selected for measurement. That would be a strange coincidence but certainly not an impossible one. You can easily check this possibility for yourself. Since the time intervals between images are equal in all cases, the speeds will be equal only if the distance intervals are equal to each other. Is the scale distance between images always 13.0 cm?

Or perhaps you are thinking, "It was rigged!" or, if you are less skeptical you may think it was just a rare event and it would not happen again. All right then, <u>you</u> try it. Most school physics laboratories have cameras, strobe lamps (or mechanical strobes, which work just as well), and low-friction discs of one sort or another. Repeat the experiment several times at different initial speeds, and then compare your results with ours.

You may have even a more serious reservation about the experiment. If you ask, "How do you know that the disc didn't slow down an amount too small to be detected by your measurements?", we can only answer that we don't. All measurements are approximations. If we had measured distances to the nearest 0.001 cm (instead of to the nearest 0.1 cm) we might have detected some slowing down. But if we again found no change in speed, you could still raise the same objection.

There is no way out of this. We must simply admit that *no* physical measurements are ever exact or infinitely precise. Thus it is fair to question any set of measurements and the findings based on them. Not only fair, but expected.

See the articles "Motion in Words" and "Representation of Movement" in *Project Physics Reader 1*.

Before proceeding further in our study of motion, let us briefly review the results of our experiment. We devised a way to measure the successive positions of a moving dry ice disc at known time intervals. From this we calculate first the distance intervals and then the speed between selected positions. We discovered that the speed did not change. Objects that move in such a manner are said to have *uniform speed*. What about *non*uniform speed? That is our next concern.

Some practice problems dealing with constant speed are given in Study Guide 1.2 (a,b,c and d).

1.4 *Leslie's "50" and the meaning of average speed*. Consider the situation at a swimming meet. As a spectator, you want to see who are the fastest swimmers in each event. At the end of each race, the name of the winner is announced, and his total time given. Speeds as such are usually not announced, but since in a given race—say the 100-yard backstroke—every swimmer goes the same distance, the swimmer with the shortest time is necessarily the one having the highest average speed. We can define average speed as follows:

$$\text{average speed} = \frac{\text{distance traveled}}{\text{elapsed time}}$$

So the speeds calculated on page 14 are all *average* speeds.

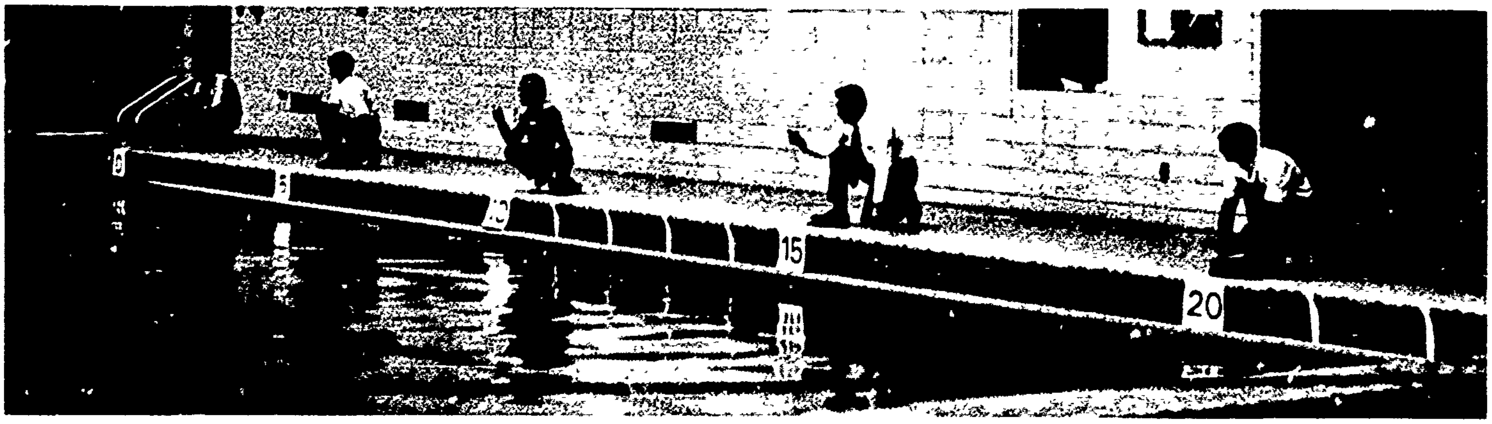
What information does a knowledge of the average speed convey? We shall answer this question by studying a real example.

Leslie is not the fastest girl freestyle swimmer in the world, but Olympic speed is not necessary for our purposes. One day after school, Leslie was timed over two lengths of the Cambridge High School pool. The pool is 25 yards long, and it took her 56.1 seconds to swim the two lengths. Thus her average speed over the 50 yards was

$$\frac{50.0 \text{ yd}}{56.1 \text{ sec}} = 0.89 \text{ yd/sec.}$$

This is the equivalent of 1.8 mph. Some speed! A sailfish can do over 40 mph, and a fin-back whale can do 20 mph. But then man is a land animal. For short distances he can run better than 20 mph. But cheetahs have been clocked at 70 mph and ostriches at 50 mph.

Did Leslie swim with uniform speed? If not, which length did she cover more quickly? What was her greatest speed? Her least? How fast was she moving when she passed the 10, or 18, or 45-yard mark? Because we do not have the answer to any of these questions, we must admit that average speed does not tell us much. All we know is that Leslie swam the 50 yards in 56.1 seconds. The number 0.89 yd/sec probably comes closer than any other *one* number to describing the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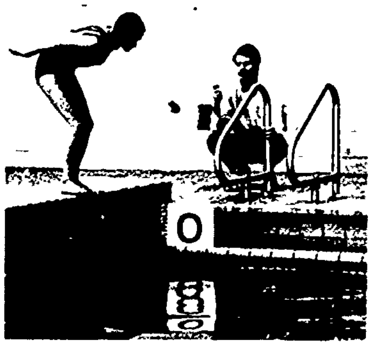
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whole event. Such a number is useful and there is no denying that it is easy to compute.

But those questions about the details of Leslie's swim still nag us. To answer them, more data are necessary. That is why we arranged the event as shown on the opposite page.

Details of the speed at different parts of a race can help athletes improve their over-all showing.

Observers stationed at 5-yard intervals from the 0 mark to the 25-yard mark started their stop watches when the starting signal was given. Each observer had two watches, one which he stopped as Leslie passed his mark going down the pool, and the other which he stopped as she passed on her return trip. The data are tabulated below.

Position (yards)	0	5	10	15	20	25	30	35	40	45	50
Time (seconds)	0.0	2.5	6.0	11.0	16.0	22.0	26.5	32.0	39.5	47.5	56.0

From these data we can determine Leslie's average speed for the first 25 yards and for the last 25 yards.

1) $$\text{Average speed for first 25 yards} = \frac{\text{distance traveled}}{\text{elapsed time}} = \frac{25 \text{ yards}}{22 \text{ seconds}} = 1.1 \text{ yds/sec.}$$

2) $$\text{Average speed for last 25 yards} = \frac{\text{distance traveled}}{\text{elapsed time}} = \frac{25 \text{ yards}}{56 \text{ sec} - 22 \text{ sec}} = \frac{25 \text{ yds}}{34 \text{ sec}} = .74 \text{ yd/sec.}$$

It is clear that Leslie did not swim with uniform speed. She swam the first length much faster (1.1 yds/sec) than the second length (0.74 yd/sec). Notice that the overall average speed (0.89 yd/sec) does not describe either lap very well. If we wish to describe Leslie's performance in more detail, it will be advantageous to modify our data table.

Before we continue our analysis of Leslie's swim, however, we shall introduce some shorthand notation. In this shorthand notation the definition of average speed can be simplified from

$$\text{average speed} = \frac{\text{distance traveled}}{\text{elapsed time}}$$

to the concise statement

$$v_{av} = \frac{\Delta d}{\Delta t} .$$

The same concepts we are here developing to discuss this everyday type of motion will be needed to discuss the motion of planets, atoms, and so forth.

In this equation v_{av} is the symbol for average speed, d is the symbol for distance, and t is the symbol for time. The symbol Δ is the fourth letter in the Greek alphabet. It is called delta. When Δ precedes another symbol, it means "the change in...." Thus, Δd does <u>not</u> mean that Δ multiplies d, but rather "the change in d" or "distance interval." Likewise, Δt stands for "change in t" or "time interval."

We can now proceed with our analysis. Suppose as a next step we calculate the average speed for each 5-yard interval. This calculation is easily done; especially when our data are organized as they are in the table below. The results of this calculation for the first lap are entered in the right-hand column.

Data Table for Leslie's 50-yard Swim

Distance (yds)	Time (sec)	Δd (yds)	Δt (sec)	Δd/Δt (yd/sec)
0	0.0	5	2.5	2.0
5	2.5	5	3.5	1.4
10	6.0	5	5.0	1.0
15	11.0	5	5.0	1.0
20	16.0	5	6.0	.8
25	22.0	5	4.5	
30	26.5	5	5.5	
35	32.0	5		
40	39.5	5		
45	47.5	5		
50	56.1			

(The second-lap computations are left to you.)

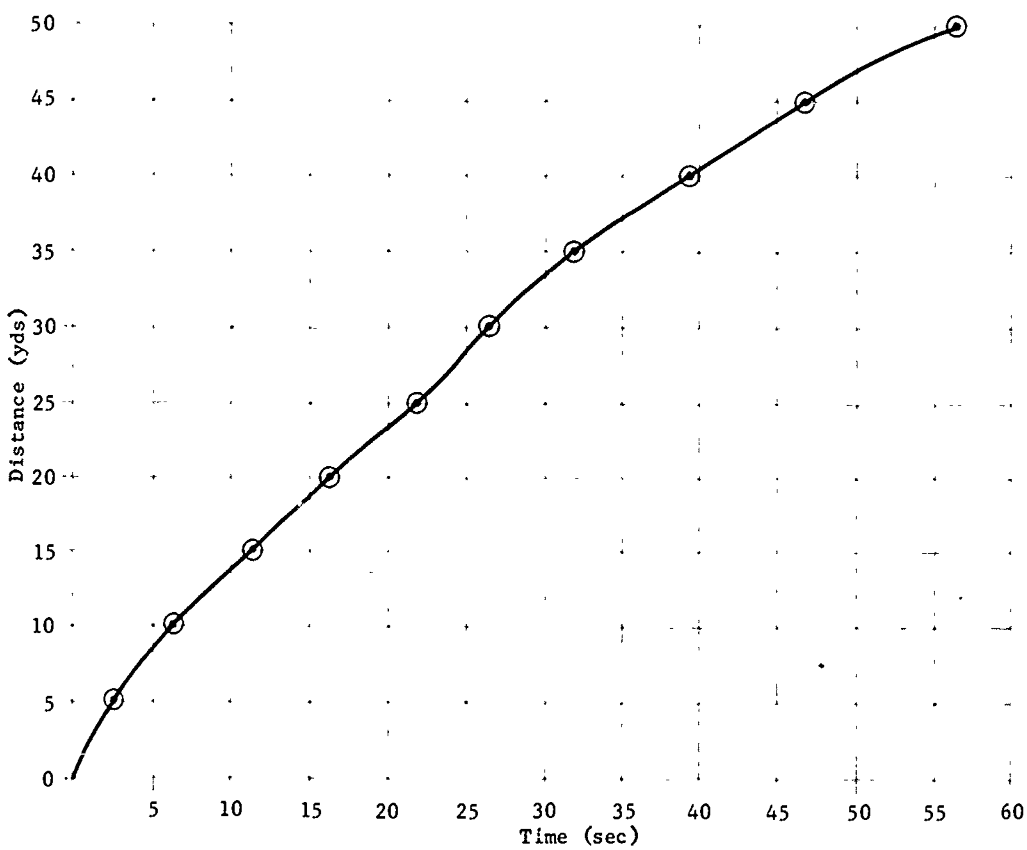
Looking at the speed column, we discover that Leslie had her greatest speed right at the beginning. During the middle part of the first length she swam at a fairly steady rate, and she slowed down coming into the turn. You can use your own figures to see what happens after the turn.

Now we have described Leslie's 50-yard swim in greater detail than when we gave a single, average speed for both lengths. But one point must be clear: although we have determined the speeds at various intervals along the path, we are still dealing with <u>average</u> speeds. The intervals are smaller—the time required to swim 5 yards rather than the entire 50—but we do not know the details of what happened within any of the intervals. Thus, Leslie's average speed between the 15 and 20-yard marks was 1.0 yd/sec, but her speed at the very instant she was 18 yards from the start is still uncertain. Even so, the average speed computed over the 15 to 20-yard interval is probably a better estimate of her speed at the 18-yard mark than the average speed computed over the whole 50 yards, or over either length. We shall come back to this problem of the determination of speed at a particular point in Sec. 1.7.

Practice problems on average speed can be found in Study Guide 1.2 (e, f, g and h). Study Guide 1.3, 1.4, 1.5 and 1.6 offer somewhat more challenging problems. Some suggestions for average speeds to measure are listed in Study Guide 1.7 and 1.8. Questions about the speedometer as a measure of speed are raised in Study Guide 1.9 and 1.10.

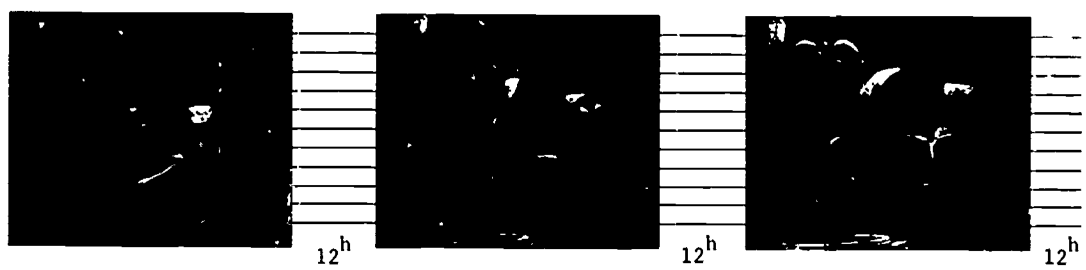
1.5 Graphing motion. What can we learn about motion by graphing data rather than just tabulating them? Let us find out by preparing a distance-versus-time graph using the data from Leslie's 50-yard swim. It is shown below. (We assumed there were no abrupt changes in her motion and so joined the data points with a smooth curve.)

Now let us "read" the graph. If you will accept the idea that the steepness of the graph in any region indicates something about the speed (the steeper the faster) you will have no trouble seeing how Leslie's speed changed throughout the trial. It will be proven to you a little later that the speed can be calculated by measuring the steepness of the graph. Notice that the graph is steepest at the start and



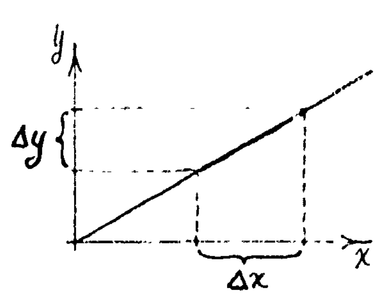
These photographs show a stormy outburst at the edge of the sun, a river of ice, and a developing sunflower plant. From these pictures and the included time intervals you can determine the average speeds (1) of the solar flare with respect to the sun's surface (radius of sun is about 432,000 mi.), (2) of the glacier with respect to the "river's bank," and (3) of the sunflower plant with respect to the flower pot.

Multiply your measurements on these photographs by 8 to get actual plant sizes.



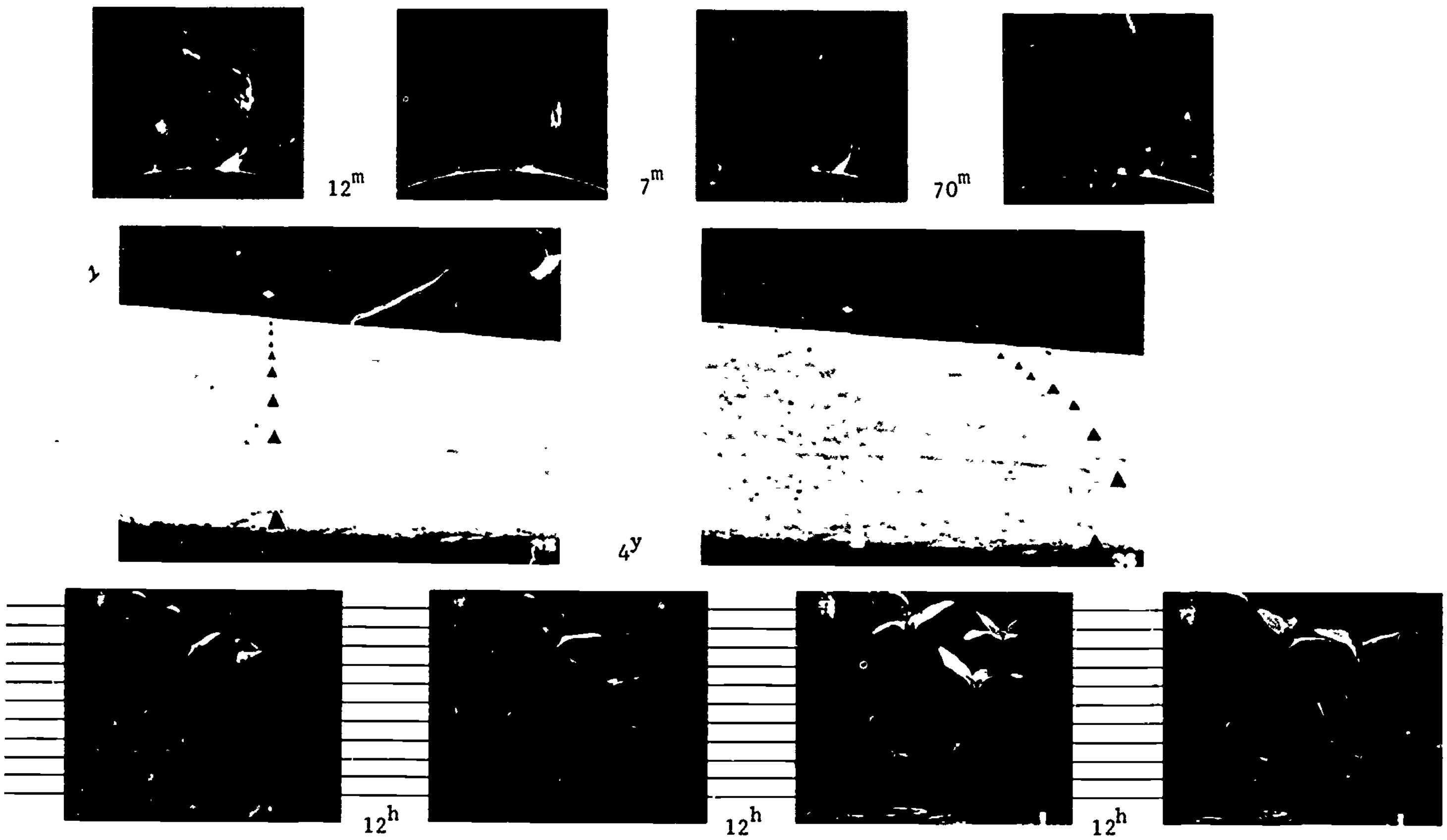
decreases slightly up to the 10-yard mark. From 10 yards to 20 yards the graph appears to be a straight line becoming neither more nor less steep. This means that her speed in this stretch neither increased nor decreased but was uniform. Reading the graph further, we see that she slowed down somewhat before she reached the 25-yard mark but gained some speed at the turn. The steepness decreases gradually from the 30-yard mark to the finish indicating that Leslie was slowing down. (She could barely drag herself out of the pool after the trial.)

Looked at in this way, a graph provides us with a picture or visual representation of motion. But our interpretation of it was merely qualitative. If we want to know just how fast or slow Leslie was swimming at various times, we need a quantitative method of expressing the steepness. The way to indicate the steepness of a graph quantitatively is by means of the "slope."



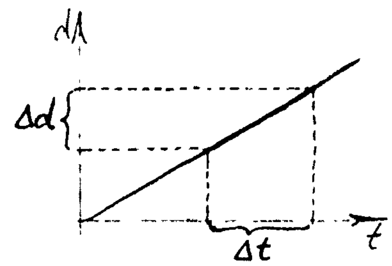
Slope is a widely used mathematical concept, and can be used to indicate the steepness in any graph. If, in accordance with custom, we call the vertical axis of any graph the y-axis and the horizontal axis the x-axis, then by definition,

$$\text{slope} = \frac{\Delta y}{\Delta x} .$$



In a distance-time graph, distance is usually plotted on the vertical axis (d replaces y) and time on the horizontal axis (t replaces x). Therefore, in such a graph,

$$\text{slope} = \frac{\Delta d}{\Delta t} .$$



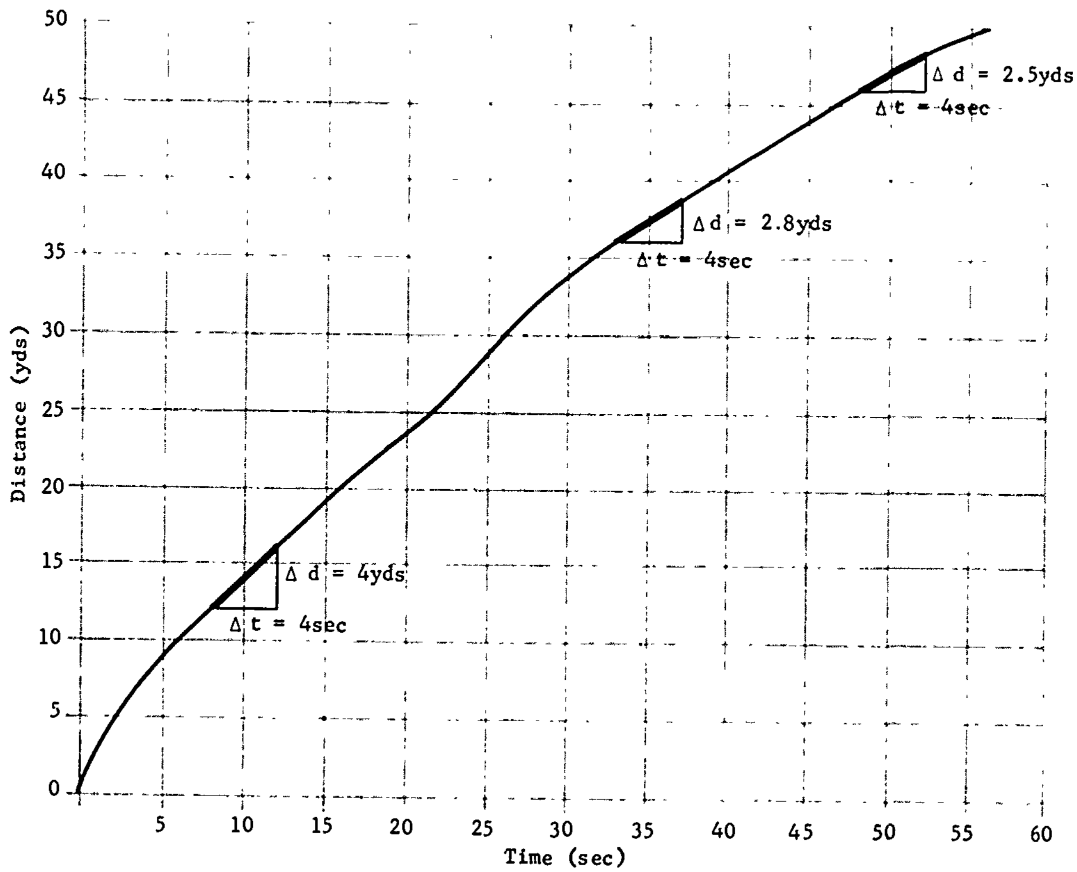
But this is just the definition of average speed. In other words, the slope of any part of a graph of distance versus time gives a measure of the average speed of the object during that interval.

There is really nothing mysterious about slope or its measurement. Highway engineers specify the steepness of a road by the slope. They simply measure the rise in the road and divide that rise by the horizontal distance one must go in order to achieve that rise. If you have never encountered the mathematical concept of slope before, or if you wish to review it, you might find it helpful to turn to Study Guide 1.11 before continuing here.

We can now ask, "What was Leslie's speed at the 14 or 47-yard marks, or at 35 seconds after the start"? In fact, by determining the slope, Leslie's speed can be estimated at <u>any</u> position or time by taking the slope of a small region on the distance-time graph of her motion that includes the particular instant or spot of interest. The answers to the above question are worked out on the graph below.

Time (sec)	Position (yds)	Speed (yds/sec)
10	11	1
20		
35		
50		

Determine Leslie's speed at these times using the graph.

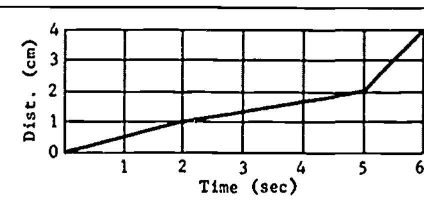


The plausibility of the results can be checked by comparing them with Leslie's average speeds near those regions. For example, her average speed during the last 10 yards (from d = 40 to d = 50) was

$$\frac{10 \text{ yards}}{56.1 \text{ sec} - 39.5 \text{ sec}} = .60 \text{ yards/sec.}$$

Similarly from the graph we determined that Leslie's speed was .62 yards/sec at the 47-yard mark.

Q1 Find the speeds at different points for a moving object from the following distance-time graph:



Q2 What was the average speed for the first 6 seconds?

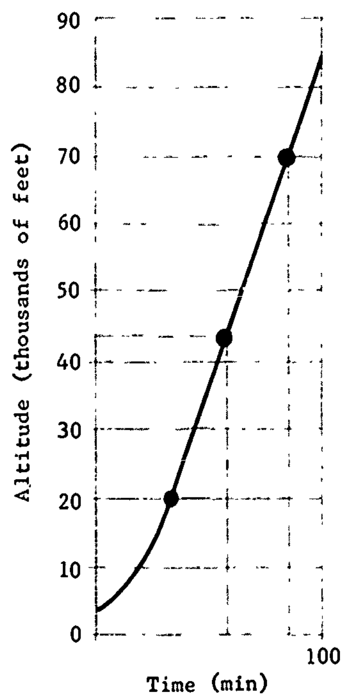
(The end-of-section questions are to help you check your understanding of the section. If your answers don't agree reasonably well with those given on pp. 127-128, you should read the section again.)

1.6 Time out for a warning. Graphs are useful—but they can also be misleading. You must always be aware of the limitations of any graph you use. The only certain pieces of information in a graph are the data points, and even they are certain only to within the accuracy limits of the measurements. Furthermore, we often lessen the accuracy when we place the points on a graph.

Turn back to p. 13 and in the margin draw a distance-time graph for the motion of the dry ice disc.

The line drawn through the points depends on personal judgment and interpretation. The process of estimating values between data points is called interpolation. That is essentially what you are doing when you draw a line between data points. Even more risky than interpolation is extrapolation, where the graph line is extended to estimate values beyond the known data.

An example of a high-altitude balloon experiment carried out in Lexington, Massachusetts, will nicely illustrate the danger of extrapolation. A cluster of gas-filled balloons carried some cosmic ray detectors high above the earth's surface, and from time to time a measurement was made of the height of the cluster. The adjoining graph shows the results for the first hour and a half. As the straight line drawn through the points suggests, the assumption is that the balloons are rising with uniform speed. Thus the speed can be calculated from the slope:



$$\text{speed of ascent} = \frac{\Delta h}{\Delta t}$$

$$= \frac{27{,}000 \text{ ft}}{30 \text{ min}}$$

$$= 900 \text{ ft/min.}$$

If you were asked how high the balloons would be at the end of the experiment (500 min), you might extrapolate, obtaining the result 500 min × 900 ft/min = 450,000 ft, which is over 90 miles high! Would you be right? Turn to Study Guide 1.13 to see for yourself.

Q3 What is the difference between extrapolation and interpolation?

1.7 Instantaneous speed. Now back to Leslie. In Sec. 1.5 we saw that distance-time graphs could be extremely helpful in describing motion. When we reached the end of the section, we were speaking of specific speeds at particular points along the path (e.g., "the 14-yard mark") and at particular instants of time (e.g., "the 35-second instant"). You might have been bothered by this, for earlier we had gone out of

1 Paris street scene, 1839

2 American street scene, 1859

Photography 1839 to the Present

Photography has an important role in our analysis of motion. These pages illustrate some of the significant advances in technique over the last century.

1 Note the lone figure in the otherwise empty street. He was getting his shoes shined. The other pedestrians did not remain in one place long enough to have their images recorded. With exposure times several minutes long the outlook for the possibility of portraiture was gloomy.

2 However, by 1859, due to improvements in photographic emulsions, illumination and lenses, it was not only possible to photograph a person at rest, but one could capture a bustling crowd of people, horses and carriages. Note the slight blur of the jaywalker's legs.

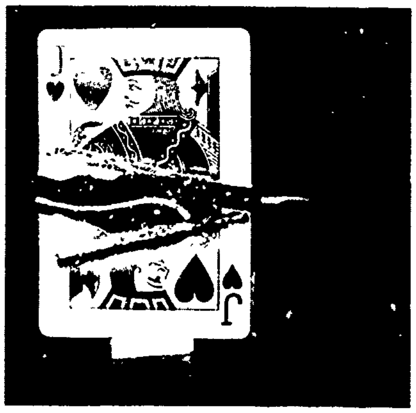
3 Today, even with an ordinary camera one can "stop" action.

4 A new medium—the motion picture. In 1873 a group of California sportsmen called in the photographer Eadweard Muybridge to settle the question, "Does a trotting horse ever have all four feet off the ground at once?" Five years later he answered the question with these photos. The six pictures were taken with six cameras lined up along the track, each camera being triggered when the horse broke a string which tripped the shutter. The motion of the horse can be reconstituted by making a flip pad of the pictures.

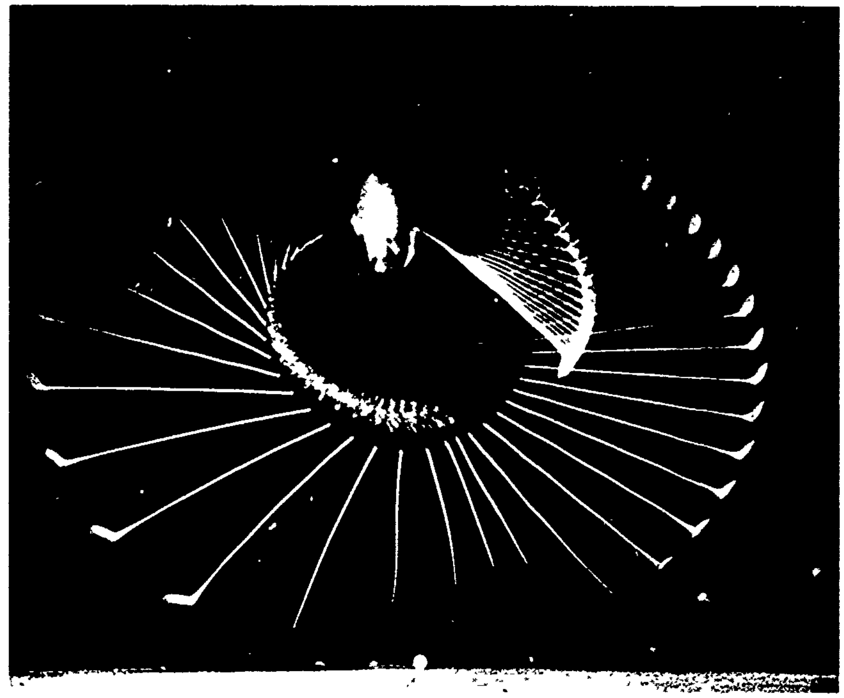
With the perfection of flexible film, only one camera was needed to take many pictures in rapid succession. By 1895, there were motion picture parlors throughout the United States. Twenty-four frames each second were sufficient to give the viewer the illusion of motion.

3 Boys on skateboards

4 Muybridge horse series, 1878



5 Bullet cutting jack of hearts, Harold Edgerton



6 Stroboscopic photo of golfer's swing, Harold Edgerton (See the article "The Dynamics of a Golf Club" in Project Physics Reader 1.)

5 It took another ninety years after the time the crowded street was photographed before a bullet in flight could be "stopped." This remarkable picture was made by Professor Harold Edgerton of MIT, using a brilliant electric spark which lasted for about one millionth of a second.

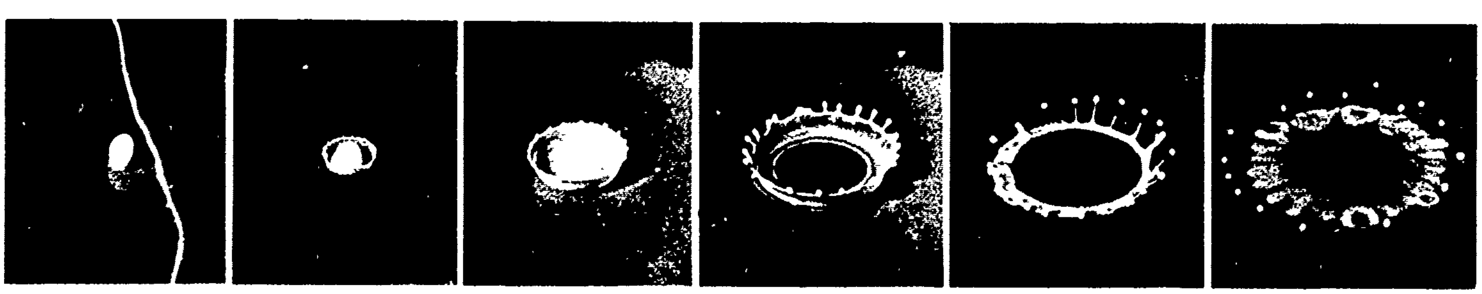
6 A light can be flashed successfully at a controlled rate and a multiple exposure (similar to the strobe photos in the book) can be made. In this photo of the golfer, the light flashed 100 times each second.

7 One does not need to have a flashing light to take multiple exposures. You can take them accidentally by forgetting to advance your film after each shot or you can do it purposely by snapping the camera shutter rapidly in succession.

8 An interesting offshoot of motion pictures is the high-speed motion picture. In the frames of the milk drop shown below, 1,000 pictures were taken each second. The film was whipped past the open camera shutter while the milk was illuminated with a flashing light (similar to the one used in photographing the golfer) synchronized with the film. When the film is projected at 24 frames each second, action which took place in 1 second is spread out over 42 seconds.

It is clear that the eye alone could not have seen the elegant details of this somewhat mundane event.

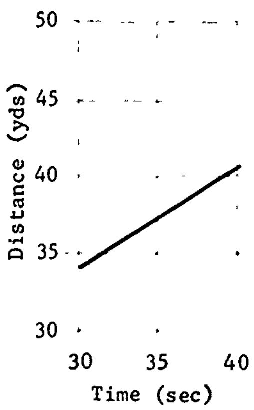
7 Girl rising



8 Action shown in high speed film of milk drop. Harold Edgerton

our way to assert that the only kind of speed we can measure is <u>average</u> speed. To find average speed we need a ratio of distance and time <u>intervals</u>; a particular point along the path does not define an interval. Nevertheless, there are grounds for stating the speed at a point. We will see what they are.

You remember that our answer to the question, "How fast was Leslie swimming at time t = 35 sec?" was "0.70 yd/sec." That answer was obtained by finding the slope of a small portion of the curve encompassing the point t = 35 sec. That section of the curve has been reproduced in the margin here. Notice that the part of the curve we used is seemingly a straight line. Thus, as the table under the graph shows, the value of the slope does not change as we decrease the time interval Δt. Now imagine that we closed in on the point where t = 35 sec until the amount of curve remaining became vanishingly small. Could we not safely assume that the slope of that infinitesimal part of the curve would be the same as that on the straight line of which it seems to be a part? We think so. That is why we took the slope of the straight line lying along the graph from t = 33.0 sec to t = 37.0 and <u>called</u> it the speed <u>at</u> t = 35.0 sec.

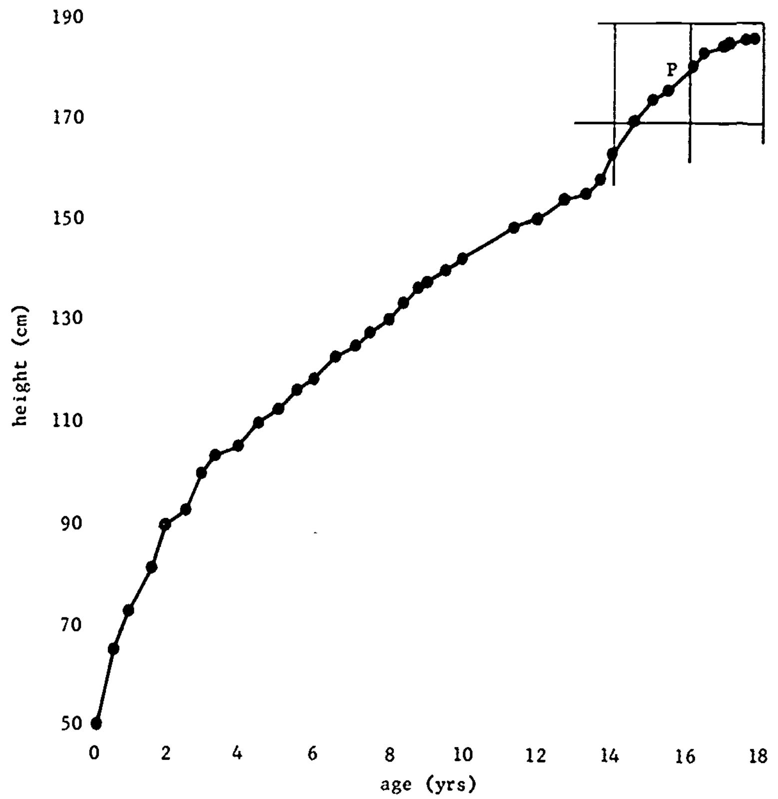


Time interval (sec)	Distance interval (yds)	$\frac{\Delta d}{\Delta t}$ (yds/sec)
33 37	36 38.8	.70
34 36	36.7 38.1	.70
34.5 35.5	37.05 37.75	.70
34.75 35.25	37.225 37.575	.70

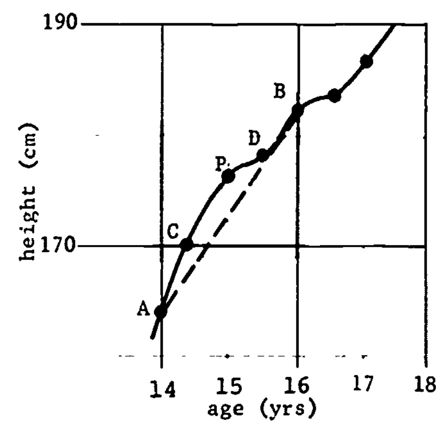
We hope you noticed that in estimating a value for Leslie's instantaneous speed at a particular time, we actually measured the average speed over a 4.0-sec interval. Conceptually, we have made a leap here. We have decided that the instantaneous speed at a particular time can be equated to an average speed $\Delta d/\Delta t$ provided: 1) that the particular time is encompassed by the time interval, Δt, used to compute $\Delta d/\Delta t$ and 2) that the ratio $\Delta d/\Delta t$ does not change appreciably as we compute it over smaller and smaller time intervals.

A concrete example will help here. In the oldest known study of its kind, the French scientist de Montbeillard periodically recorded the height of his son during the period 1759-1777. A graph of height versus age is shown on the next page.

From the graph we can compute the boy's average growth rate over the entire 18-year interval or over any other time interval. Suppose, however, we wanted to know how fast the boy was growing on his fifteenth birthday. The answer becomes evident if we enlarge the graph in the vicinity of the fifteenth year. His height at age 15 is indicated as point P, and the other letters designate time intervals on either side of P. The boy's average growth rate over a two-year interval is given by the slope AB. Over a one-year interval this average growth rate is given in the slope DC. The slope of EF gives the average growth rate over six months,etc. The three lines are not quite parallel to each other and so their slopes will be different. In the enlarged sections below, lines have been drawn joining the end points of time intervals of 4 mo, 2 mo and 1 mo around the point t = 15 years.



Notice that for intervals less than t = 1 yr, the lines appear to be parallel to each other and gradually to merge into the curve, becoming nearly indistinguishable from it. You can approximate the tangent to this curve by placing a ruler along the line GH and extending it on both sides.

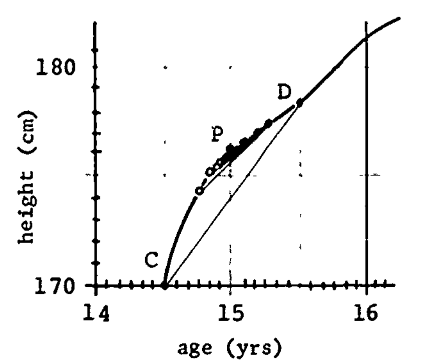


The graph above is an enlargement of the corner of the graph at the top. The graph below is a further enlargement of the middle of the enlargement.

The values of the slopes have been computed for the several time intervals and are tabulated below.

Δt	Δd	v
8 yr	49.0 cm	6.1 cm/yr
2 yr	19.0 cm	9.5 cm/yr
1 yr	8.0 cm	8.0 cm/yr
6 mo	3.5 cm	7.0 cm/yr
4 mo	2.0 cm	6.0 cm/yr
2 mo	1.0 cm	6.0 cm/yr

We note that the values of v_{av} calculated for shorter and shorter time intervals approach closer and closer to 6.0 cm/yr. In fact, for any time interval less than 2 months, the average speed v_{av} will be 6.0 cm/yr within the limits of accuracy of the measurement of d and t. Thus, we can say that on young de Montbeillard's fifteenth birthday, he was growing at a rate of 6.0 cm/yr.



Average speed, we have said, is the ratio of distance traveled to elapsed time, or, in symbols,

$$v_{av} = \frac{\Delta d}{\Delta t}.$$

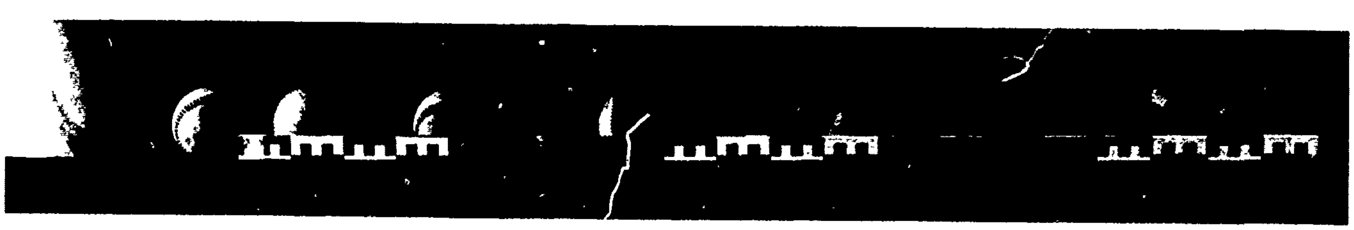
We now define instantaneous speed at a point in time as the limiting value approached by the average speeds in time-intervals including that point, as Δt is made smaller and smaller. In almost all physical situations such a limiting value can be accurately estimated by the method described on the previous page. From now on we will use the letter v, without any subscript, to mean the instantaneous speed defined in this way. (For further discussion, see the article "Speed" in Project Physics Reader 1.)

Why this definition of instantaneous speed? We can, of course, define it any way we please; whether the definition is a wise one is a matter of how useful it turns out to be in analyzing motion. In chapter 3 we will find that change of instantaneous speed, defined in this way, is related in a beautifully simple way to force.

You may be wondering why we have used the letter "v" instead of "s" for speed. The word "velocity" is often used to mean the same thing as speed. In physics it is useful to reserve "velocity" for the concept of speed in a specified direction, and denote it by the symbol $\vec{v}$. When the direction is not specified, we remove the arrow and just use the letter v, calling it speed. This distinction between v and $\vec{v}$ will be discussed in more detail in Section 3.2.

Q4 Explain the difference between average speed and instantaneous speed.

Q5 The baseball shown in the figure below is presented here for your analysis. You might tabulate your measurements and construct a distance-time graph. From the distance-time graph, you can determine the instantaneous speed at several times and construct a speed-time graph. The time interval between successive flashes is 0.5 sec. You can check your results by referring to the answer page at the back of this unit.



1.8 Acceleration—by comparison. The baseball in the problem above was changing speed—accelerating. You could tell that its speed was changing without having to take measurements and plot graphs. But how would you describe how fast the ball was changing speed?

To answer this question you have really only one new thing to learn—the definition of acceleration. Actually, the definition is simple, so the problem is not so much for you to learn it as it is to learn how to use it in situations like the one above. For the time being we will define the time-rate of change of speed as acceleration. Later, this definition will have to be modified somewhat when we encounter motion in which change in direction becomes an important factor. But for now, as long as we are dealing only with straight line motion, we can equate the time-rate of change of speed with acceleration.

Many of the effects of acceleration are well known to us. It is acceleration, not speed, that we feel when an elevator starts up or slows down. The sudden flutter in our stomachs comes only during the speeding up and slowing down portions of the trip, and not during most of the ride when the elevator is moving at a steady speed. Likewise, much of the excitement of the roller coaster and other rides at amusement parks is directly related to their unexpected accelerations. How do you know it is really not speed that causes these sensations? Simply stated, you always detect speed by reference to objects outside yourself. You can only tell you are moving at a high speed in an automobile by watching the scenery as it whizzes past you, or by listening to the sounds of air rushing against the car or the whine of the tires on the pavement. In contrast, you "feel" accelerations and do not need to look out your car window to realize the driver has stepped on the accelerator or slammed on the brakes.

Now let us compare acceleration and speed:

The rate of change of position is speed.	The rate of change of speed is acceleration.

This similarity of form will enable us to use our previous work on the concept of speed as a guide for making use of the concept of acceleration. The techniques which you have already learned for analyzing motion in terms of speed can be used to study motion in terms of acceleration. For example you have learned that the slope of a distance-time graph at a point is the instantaneous speed. What would the slope (i.e., $\Delta v/\Delta t$) of a speed-time graph indicate?

The remainder of this section is made up of a list of statements about motion along a straight line. The list has two purposes: 1) to help you review some of the main ideas about speed presented in this chapter, and 2) to present the corresponding ideas about acceleration so you may take advantage of your knowledge of speed. For this reason, each statement about speed is immediately followed by a parallel statement about acceleration.

For example, if speed changes from 4 m/sec to 5 m/sec during an interval of 1 second, average acceleration is 1 (m/sec)/sec. This is usually written more briefly as 1 m/sec^2.

1. Speed is the rate of change of position. Acceleration is the rate of change of speed.

2. Speed is expressed in the units distance/time. Acceleration is expressed in the units speed/time.

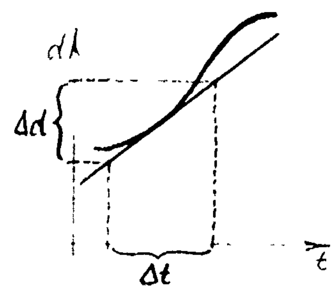
3. Average speed over any interval is the ratio of the corresponding distance and time intervals:

$$v_{av} = \frac{\Delta d}{\Delta t} .$$

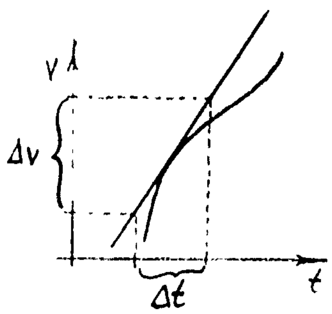
An airplane changes its speed from 350 mi/hr to 470 mi/hr in 6.0 min. Its average acceleration is 20 (mi/hr)/min—whether or not the acceleration is uniform.

Average acceleration over any interval is the ratio of the corresponding speed and time intervals:

$$a_{av} = \frac{\Delta v}{\Delta t} .$$



$$V = \frac{\Delta d}{\Delta t}$$



$$a = \frac{\Delta v}{\Delta t}$$

4. Instantaneous speed is the value approached by the average speed as Δt is made smaller and smaller. Instantaneous acceleration is the value approached by the average acceleration as Δt is made smaller and smaller.

5. If a distance-time graph is made of the motion of an object, the instantaneous speed at any position will be given by the slope of the tangent to the curve at the point of interest. If a speed-time graph is made of the motion of an object, the instantaneous acceleration at any position will be given by the slope of the tangent to the curve at the point of interest.

In this listing of statements about speed and acceleration, the concepts of average and instantaneous acceleration have been included for the sake of completeness. However, it will be helpful to remember that when the acceleration is uniform, it can be found by using the relationship

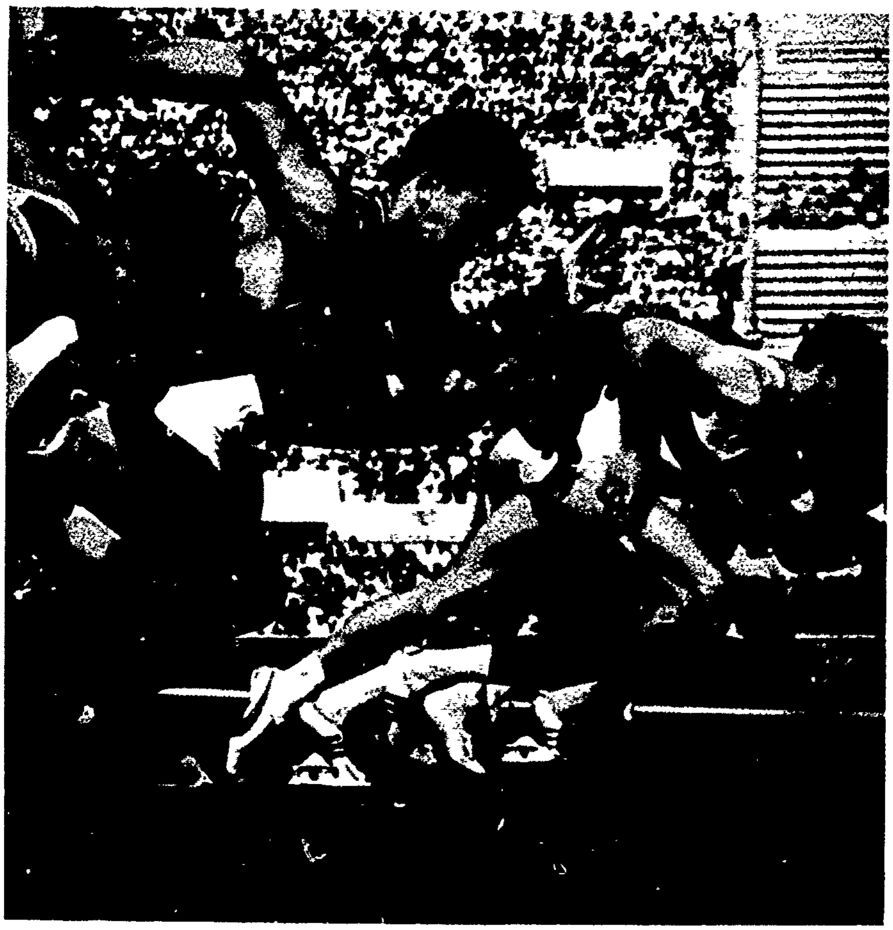
$$a = \frac{\Delta v}{\Delta t}$$

for any interval whatever. That is, instantaneous and average acceleration have the same numerical value for constant acceleration—which will be the most usual case of motion we shall encounter.

Until the work of Galileo in the seventeenth century, acceleration proved to be a particularly difficult concept. In the next chapter, we will examine Galileo's contribution to our understanding of the nature of accelerated motion. His work provides a good example of how scientific theory and actual measurements are combined to develop physical concepts.

Q6 What is the average acceleration of a sports car which goes from 0 to 60 mph in 5 seconds?

Q7 What is your average acceleration if you change your speed from 4 miles per hour to 2 miles per hour in an interval of 15 minutes?



1.1 This book is probably different in many ways from textbooks you have had in other courses. Therefore we feel it might help to make a few suggestions concerning how to use it.

1. Unless you are told otherwise by your teacher you should feel free to write in the book. Indeed we encourage you to do so. You will note that there are wide margins. One of the purposes of leaving that much space is to enable you to write down questions or statements as they occur to you as you are studying the material. Mark passages that you do not understand so that you can seek help from your teacher. You also notice that from time to time tables are left incomplete or problems appear in the text or margin. Complete such tables and write your answers to problems right in the text at the point where they are raised.

2. You will find answers to all of the end-of-section review questions on page 127, and brief answers to some of the Study Guide Questions on page 129. Always try to do the problems yourself first and then check your answers. If your answer agrees with the one in the book, then it is a good sign that you understand the material (although it is true, of course, that you can sometimes get the right answer for the wrong reason).

3. There are many different kinds of items in the Study Guide at the end of each chapter. It is not intended that you should do everything there. Sometimes we put into the Study Guide material which we think will interest some students but not enough students to merit putting into the main part of the text. Notice also that there are several kinds of problems. Some are intended to give practice and help the student in learning a particular concept whereas others are designed to help you bring together several related concepts. Still other problems are intended to challenge those students who like numerical problems.

4. Activities and experiments which you can carry out at home or outside the laboratory are described. We do not suppose that you want to do all of these but we do want you to take them seriously. If you do you will find that you are able to do quite a bit of science without having to have an elaborate laboratory.

5. The Project Physics course includes many other materials in addition to this book, such as film loops, programmed instruction booklets, and transparencies. Be sure to familiarize yourself with the Student Handbook, which describes further outside activities as well as laboratory experiments, and the Reader, which contains interesting articles related to physics.

1.2 Some practice problems:

	Situation	Find	Do work
a	Speed uniform, distance = 72 cm, time = 12 sec	Speed	
b	Speed uniform at 45 mph	Distance traveled in 20 min	
c	Speed uniform at 36 ft/min	Time to move 9.0 ft	
d	$d_1 = 0, s_2 = 15$m, $d_3 = 30$ m $t_1 = 0,$ $t_2 = 5.0$ sec, $t_3 = 10$ sec	Speed and position at 8.0 sec	
e	You drive 240 mi in 6.0 hr	Average speed	
f	Same	Speed and position after 3.0 hr	
g	Average speed = 76 cm/sec computed over a distance of 418 cm	Time taken	
h	Average speed = 44 m/sec computed over time interval of 0.20 sec	Distance moved	

1.3 If you traveled one mile at a speed of 1000 miles per hour and another mile at a speed of 1 mile per hour your average speed would not be $\frac{1000 + 1}{2}$ mph or 500.5 mph. What would be your average speed?

1.4 A tsunami (incorrectly called "tidal wave") caused by an earthquake occurring near Alaska in 1946 consisted of several sea waves which traveled at the average speed of 490 miles/hour. The first of the waves reached Hawaii four hours and 34 minutes after the earthquake occurred. From these data, calculate how far the origin of the tsunami was from Hawaii.

1.5 Light and radio waves travel through a vacuum in a straight line at a speed of nearly 3×10^8 m/sec. The nearest star, Alpha Centauri, is 4.06×10^{16} m distant from us. If this star possesses planets on which highly intelligent beings live, how soon could we expect to receive a reply after sending them a radio or light signal strong enough to be received there?

1.6 What is your average speed in the following cases:

a) You run 100 m at a speed of 5.0 m/sec and then you walk 100 m at a speed of 1.0 m/sec.
b) You run for 100 sec at a speed of 5.0 m/sec and then you walk for 100 sec at a speed of 1.0 m/sec?

1.7 Design some experiments which will enable you to make estimates of the average speeds for some of the following objects in motion.

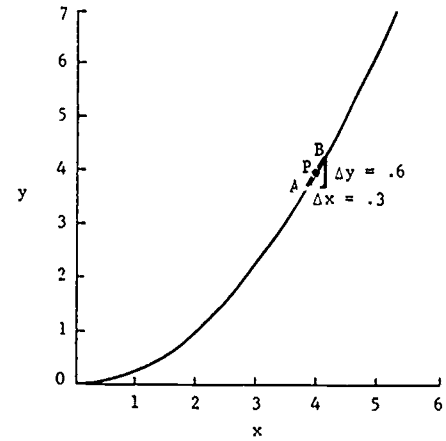
a) Baseball heaved from outfield to home plate
b) The wind
c) A cloud
d) A raindrop (do all drops have different speeds?)
e) Hand moving back and forth as fast as possible
f) The tip of a baseball bat
g) Walking on level ground, up stairs, down stairs
h) A bird flying
i) An ant walking
j) A camera shutter opening and closing

1.8 What problems arise when you attempt to measure the speed of light? Can you design an experiment to measure the speed of light?

1.9 Sometime when you are a passenger in an automobile compare the speed as read from the speedometer to the speed calculated from $\Delta s/\Delta t$. Explain any differences.

1.10 An automobile speedometer is a small current generator driven by a flexible cable run off the drive shaft. The current produced increases with the rate at which the generator is turned by the rear axle. The speedometer needle indicates the current. Until the speedometer is calibrated it can only indicate changes in speed, but not actual speeds in miles per hour. How would you calibrate the speedometer in your car if the company had forgotten to do the job? If you replaced your 24" diameter rear wheels with 28" diameter wheels, what would your actual speed be if your speedometer read 50 mph? Would your speedometer read too high or too low if you loaded down the rear end of your car and had the tire pressure too low? What effect does the speedometer have on the speed of the car? Can you invent a speedometer that has no effect on the motion of the car?

1.11 Take a look at the graph of y versus x shown below:



Notice that in this graph the steepness increases as x increases. One way to indicate the steepness of the graph at a point is by means of the "slope." The numerical value of the slope at a point P is obtained by the following procedure, which is diagramed above. Move a short distance along the graph from point A to point B, which are on the curve and lie on either side of point P. Measure the change in y, (Δy) in going from A to B. In this example $\Delta y = .6$. Measure the corresponding change in x, (Δx) in going from A to B. Δx here is .3. The slope is defined as the ratio of Δy to Δx.

$$\text{Slope} = \frac{\Delta y}{\Delta x}.$$

In the example

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{.6}{.3} = 2.$$

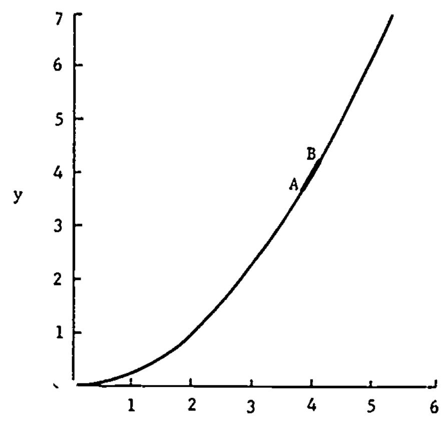
Now there are three important questions concerned with slopes that we must answer.

Q. What are the dimensions or units for the slope?

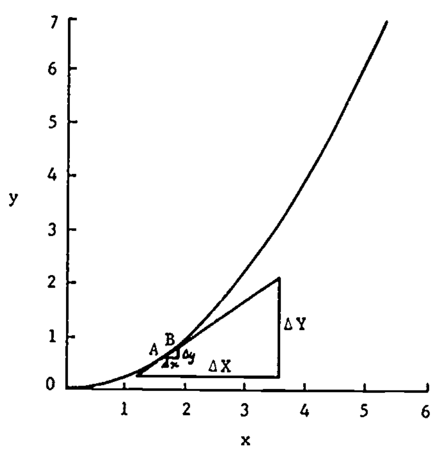
A. The dimensions are just those of y/x. For example, if y represents a distance in meters and x represents a time in seconds then the units for slope will be meters/seconds or meters per second.

Q. In practice how close do A and B have to be to point P? (Close is not a very precise adjective. New York is close to Philadelphia if you are traveling by jet. If you are walki ~ it is not close.)

A. Choose A and B near enough to point P so that the line connecting A and B lies along the curve at point P. For example:



Q. Suppose A and B are so close together that you cannot read Δx or Δy from your graph. What does one do to calculate the slope?

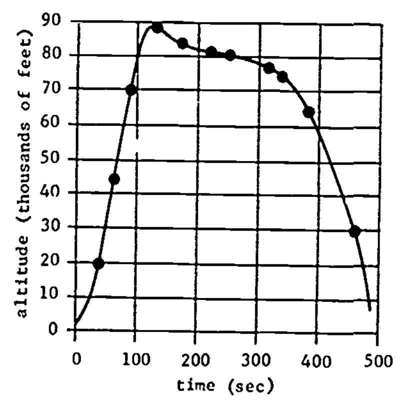


A. Extend line AB as it is shown in the figure and compute its slope. Notice that the small triangle is similar to the large triangle and that $\frac{\Delta Y}{\Delta X} = \frac{\Delta y}{\Delta x}$.

Determine the slopes of this graph of distance versus time at t = 1, 2, 3 and 4 seconds.

1.12 The electron beam in a TV set sweeps out a complete picture in 1/30th of a second and each picture is composed of 525 lines. If the width of the screen is 20 inches, what is the speed of that beam over the surface of the screen?

1.13 (Answer to question in text, page 23.) Indeed the prediction based upon the first $1\frac{1}{2}$ hour was vastly wrong. Such a prediction, based on a drastic extrapolation from the first $1\frac{1}{2}$ hour's observation, neglects all the factors which limit the maximum height obtainable by such a cluster of balloons, such as the bursting of some of the balloons, the change in air pressure and density with height, etc. In fact, at the end of 500 minutes, the cluster was not 450,000 feet high, but had come down again, as the distance-time graph for the entire experiment shows. For another extrapolation problem, see Study Guide 1.14.



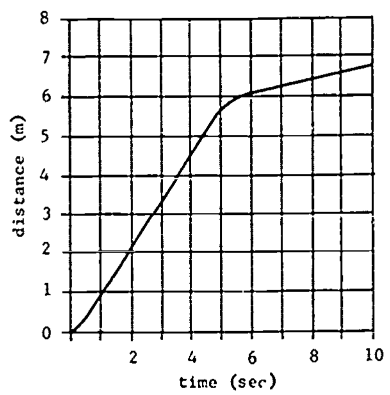
The altitude of a cosmic ray detector carried aloft by a cluster of balloons.

1.14 World's 400-meter swimming records for men and women. Ages are in parentheses:

1926	4:57.0	Weissmuller (18)
	5:53.2	Gertrude Ederle (17)
1936	4:46.4	Syozo Makino (17)
	5:28.5	Helene Madison (18)
1946	4:46.4	Makino (17)
	5:00.1	Hveger (18)
1956	4:33.3	Hironoshin Furuhashi (23)
	4:47.2	Crapp (18)
1966	4:11.1	Frank Weigand (23)
	4:38.0	Martha Randall (18)

By about how many meters would Martha Randall have beaten Johnny Weissmuller if they had raced each other? Could you predict the 1976 world's record for the 400-meter race by extrapolating the graph of world records vs. dates up to the year 1976?

1.15

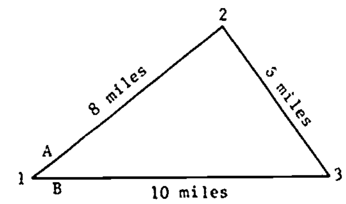


Detailed analysis of a stroboscopic photograph of a rolling ball yielded information which was plotted on the graph above. By placing your ruler tangent to the curve at appropriate points estimate the following:

a) At what moment or interval was the speed greatest? What was the value of the speed at that time?
b) At what moment or interval was the speed least? What was it at that time?
c) What was the speed at time 5.0 sec?
d) What was the speed at time 0.5 sec?
e) How far did the ball move from time 7.0 sec to 9.5 sec?

1.16 Suppose you must measure the instantaneous speed of a bullet as it <u>leaves</u> the barrel of a rifle. Explain how you would do this.

1.17



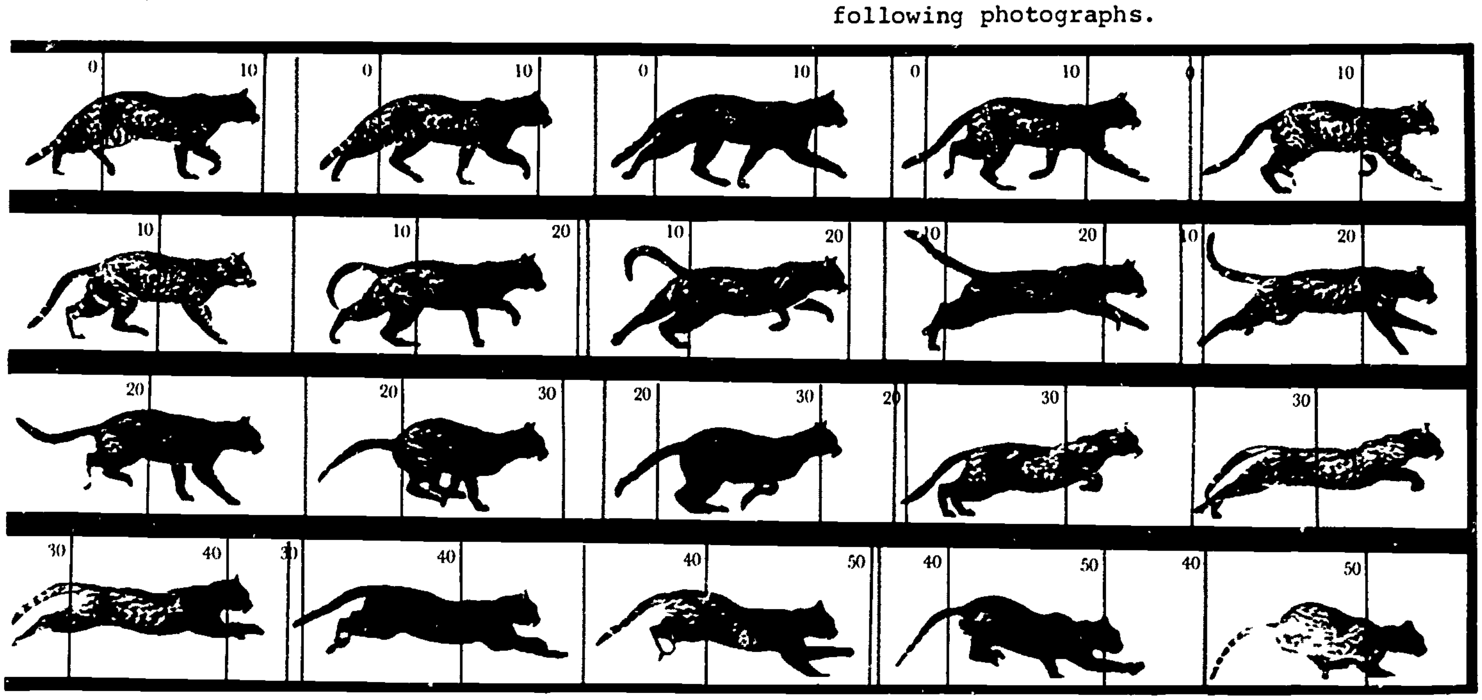
Car A and car B leave point 1 simultaneously and both travel at the <u>same speed</u>. Car A moves from 1 to 2 to 3 while car B moves from 1 to 3 directly. If B arrives at point 3 six minutes before A arrives, what was the speed of either car?

1.18 The data below show the instantaneous speed in a test run of a Corvette car, starting from rest. Plot the speed-versus-time graph, and derive and plot the acceleration-time graph.

a) What is the speed at t = 2.5 sec?
b) What is the maximum acceleration?

Time (sec)	Speed (m/sec)
0.0	0.0
1.0	6.3
2.0	11.6
3.0	16.5
4.0	20.5
5.0	24.1
6.0	27.3
7.0	29.5
8.0	31.3
9.0	33.1
10.0	34.9

1.19 Discuss the motion of the cat in the following photographs.



The numbers on each photograph indicate the number of inches measured from the line marked "0"

Chapter 2 Free Fall-Galileo Describes Motion

Portrait of Galileo in crayon by Ottavio Leoni, a contemporary of Galileo.

2.1 The Aristotelian theory of motion. In this chapter we shall take a look at an important piece of research: Galileo's study of freely falling bodies. While the physical problem of free fall is fascinating in itself, our emphasis will be on Galileo as one of the first modern scientists. Thus Galileo's view of the world, his way of thinking, his use of mathematics and his reliance upon experimental tests are as important to us as the actual results of his investigation.

To understand the nature of Galileo's work and to appreciate its significance, we must first examine the differences between Galileo's new science of physics and the medieval system of physical thought that it eventually replaced. By comparing the new with the old, we can see how Galileo helped change our way of thinking about the world.

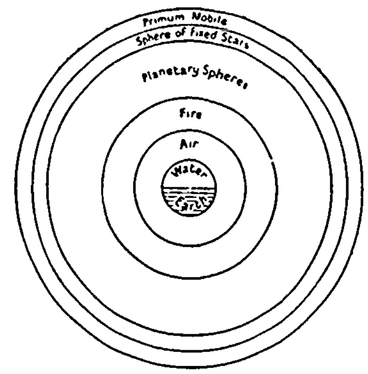


Diagram of medieval concept of the world structure.

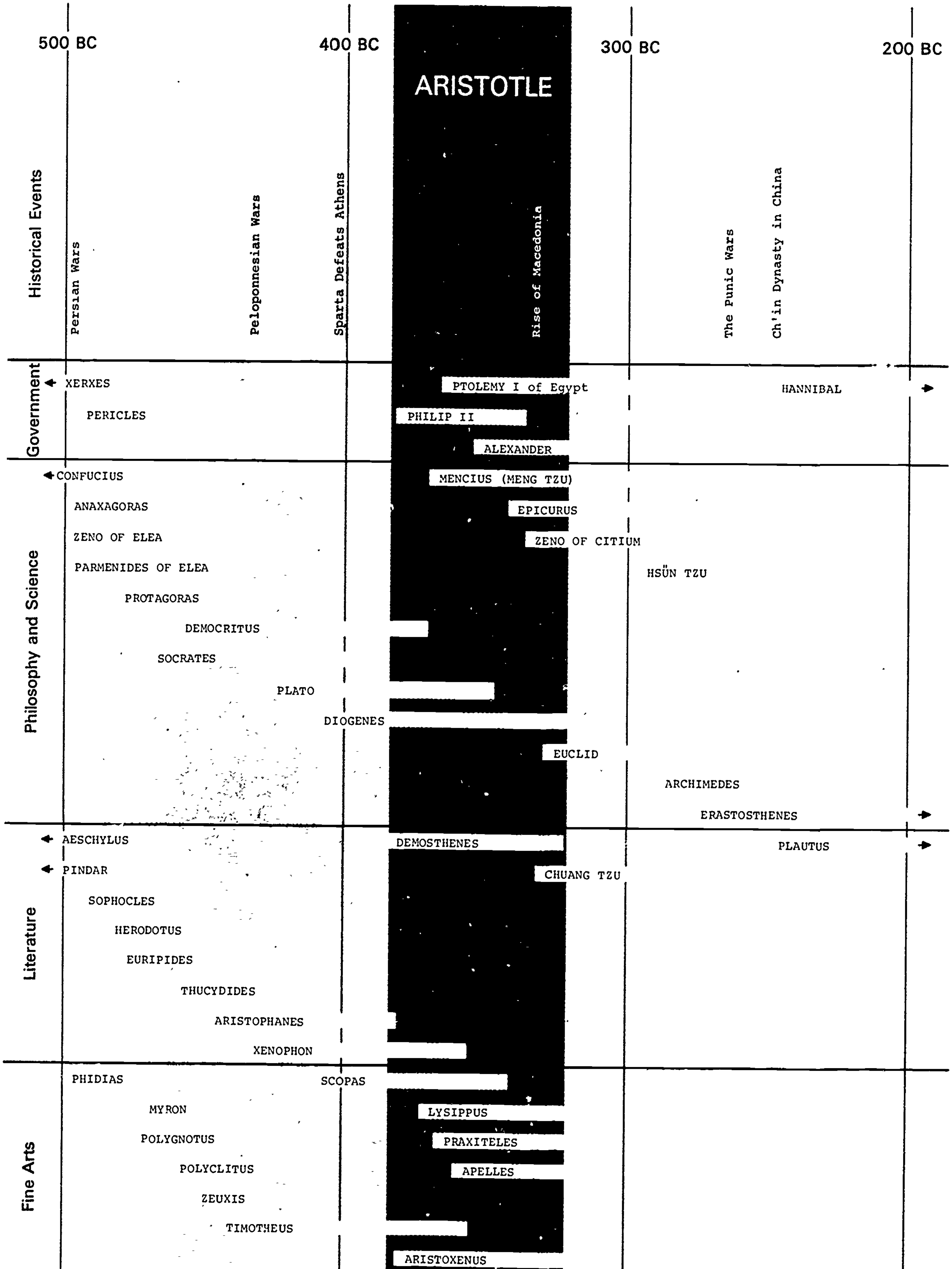
In medieval physical science, as Galileo learned it at the University of Pisa, there was a sharp distinction between the objects on the earth and those in the sky. All terrestrial matter, the matter within our physical reach, was believed to be a mixture of four "elements"—Earth, Water, Air and Fire. Each of these four elements was thought to have a natural place in the terrestrial region. The highest place was allotted to Fire. Beneath Fire was Air, then Water and, finally, in the lowest position, Earth. Each was thought to seek its own place. Thus, Fire would tend to rise through Air, and Air through Water, whereas Earth would tend to fall through both Air and Water. The actual movement of any real object depended on the particular mixture of these four elements making it up and where it was in relation to its natural place.

A good deal of commonsense experience supports this view. For example, Water bubbles up through Earth at springs. When sufficient Fire is added to ordinary Water, by heating it, the resulting mixture of elements (what we call steam) rises through the air. Can you think of other examples?

The medieval thinkers also believed that the stars, planets and other celestial bodies moved in a far simpler manner than those objects on, or near, the earth. The celestial bodies were believed to contain none of the four ordinary elements, but instead consisted solely of a fifth element, the quintessence. The natural motion of objects composed of this element was neither rising nor falling, but endless revolution in circles around the center of the universe. The center of the universe was considered to be identical with the center of the Earth. Heavenly bodies, although moving, were thus at all times in their natural places. They were thus set apart from terrestrial objects, which displayed natural motion only as they returned to their natural places from which they were displaced.

From quinta essentia meaning fifth essence. In earlier Greek writings the term for it was ether.

This theory, so widely held in Galileo's time, originated in the fourth century B.C.; we find it mainly in the writings


500 BC
400 BC
ARISTOTLE
300 BC
200 BC
Historical Events
Persian Wars
Peloponnesian Wars
Sparta Defeats Athens
Rise of Macedonia
The Punic Wars
Ch'in Dynasty in China
Government
XERXES
PERICLES
PTOLEMY I of Egypt
PHILIP II
ALEXANDER
HANNIBAL
Philosophy and Science
CONFUCIUS
ANAXAGORAS
ZENO OF ELEA
PARMENIDES OF ELEA
PROTAGORAS
DEMOCRITUS
SOCRATES
PLATO
DIOGENES
MENCIUS (MENG TZU)
EPICURUS
ZENO OF CITIUM
HSÜN TZU
EUCLID
ARCHIMEDES
ERASTOSTHENES
Literature
AESCHYLUS
PINDAR
SOPHOCLES
HERODOTUS
EURIPIDES
THUCYDIDES
ARISTOPHANES
XENOPHON
DEMOSTHENES
CHUANG TZU
PLAUTUS
Fine Arts
PHIDIAS
MYRON
POLYGNOTUS
POLYCLITUS
ZEUXIS
TIMOTHEUS
SCOPAS
LYSIPPUS
PRAXITELES
APELLES
ARISTOXENUS

of the Greek philosopher Aristotle. A physical science of order, rank and place, it fits well many facts of everyday observation. Moreover, these conceptions of matter and motion were part of an all-embracing scheme or "cosmology" by which Aristotle sought to relate ideas which are nowadays discussed separately under the headings of science, poetry, politics, ethics and theology.

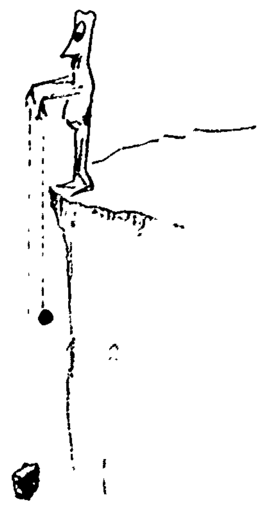
Aristotle was born in 384 B.C. in Stageira, a city in the Greek province of Macedonia. His father was the physician to Amyntas II, the king of Macedonia, and so Aristotle's early childhood was spent in an environment of court life. At the age of 17 he was sent to Athens to complete his education. He spent 20 years there, first as a student and then as a colleague of Plato. When Plato died, Aristotle left Athens and later returned to Macedonia to become the private tutor of Alexander the Great (356-323 B.C.). In 335 B.C., Aristotle came back to Athens and founded the Lyceum, a school and center of research. Little is known of his physical appearance and little biographical information has survived. Fortunately, 50 volumes of his writings (out of perhaps 400 in all) did survive. These works of Aristotle remained unknown in Western Europe for 1500 years after the decline of the ancient Greek civilization, until they were rediscovered in the thirteenth century A.D. and incorporated into Christian theology. Aristotle became such a dominant influence in the late Middle Ages that he was referred to simply as "The Philosopher."

Because of his habit of lecturing in the walking place (peripatos, in Greek) of the Lyceum, Aristotle's company of philosophers came to be known as the "Peripatetics."

This painting titled "School of Athens" was done by Raphael in the beginning of the sixteenth century. The painting clearly reflects one aspect of the Renaissance, a rebirth of interest in classical Greek culture. The central figures are Plato (on left) and Aristotle. Raphael used Leonardo da Vinci as his model for Plato.

The works of Aristotle constitute an encyclopedia of ancient Greek thought—some of it summarized from the work of others, but much of it created by Aristotle himself. Today it seems incredible that one man could have written so intelligently and knowledgeably on such different subjects as logic, philosophy, theology, physics, astronomy, biology, psychology, politics and literature. Some scholars doubt it was all the work of one man.

Unfortunately, Aristotle's physical theories had limitations which became evident much later, and we will devote part of this chapter to showing where these limitations lie in some specific cases. But this should not detract from Aristotle's great achievements in other fields.



Aristotle: rate of fall is proportional to weight divided by resistance.

According to Aristotle, the fall of a heavy object toward the center of the earth is a natural motion. What factors determine the rate of fall? A rock falls faster than a leaf; therefore, he reasoned, weight must be a factor. An object falls faster in air than in water, so the resistance of the medium must also be a factor. Other factors, such as the color and temperature of the object, could conceivably affect the rate of fall, but to Aristotle these were evidently of little importance. He assumed that the rate of fall must therefore increase in proportion to the weight of the object, and decrease in proportion to the resisting force of the medium. The actual rate of fall in any particular case would be determined by dividing the weight by the resistance. In his book On the Heavens, Aristotle makes the following statement about natural motion (such as falling):

> A given weight moves a given distance in a given time; a weight which is heavier moves the same distance in less time, the time being inversely proportional to the weights. For instance, if one weight is twice another, it will take half as long over a given distance.

See Study Guide 2.13.

Aristotle also discussed "violent" motion—that is, motion of an object which is not toward its natural place. Such motion, he argued, must always be caused by a force, and the speed of the motion will increase as the force increases. When the force is removed, the motion must stop. This theory agrees with our common experience in pushing desks or tables across the floor. It doesn't seem to work quite so well for objects thrown through the air, since they keep moving for a while even after we have stopped exerting a force on them. To account for this kind of motion, Aristotle assumed that the air itself somehow exerts a force that continues to propel an object moving through it.

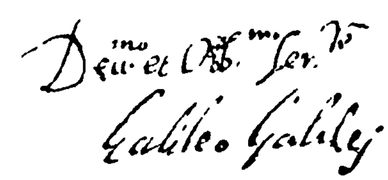
John Philoponus: rate of fall is proportional to weight minus resistance.

Later scientists proposed some modifications in Aristotle's theory of motion. For example, John Philoponus of Alexandria, in the fifth century A.D., argued that the speed of an object in natural motion should be found by subtracting the resistance of the medium from the weight of the object, rather than dividing by the resistance. Philoponus claimed that he had actually done experiments to support his theory, though he did not report all the details; he simply said that he dropped two weights, one of which was twice as heavy as the other, and observed that the heavy one did not reach the ground in half the time taken by the light one.

There were still other difficulties with Aristotle's theory of motion. However, the realization that his teachings concerning motion had their limitations did little to modify the important position given to them in the universities of France and Italy during the fifteenth and sixteenth centuries. In any case, the study of motion through space was of major interest to only a few scholars and, indeed, it had been only a very small part of Aristotle's own work. Nevertheless, Aristotle's theory of motion fitted much of human experience in a general—if qualitative—way.

Two further influences stood in the way of radical changes in the theory of motion. First, Aristotle had believed that mathematics was of little value in describing change. Second, he had put great emphasis upon qualitative observation as the basis for all theorizing. Simple qualitative observation was very successful in Aristotle's biological studies. But progress in physics began only when careful measurements were made under controlled conditions.

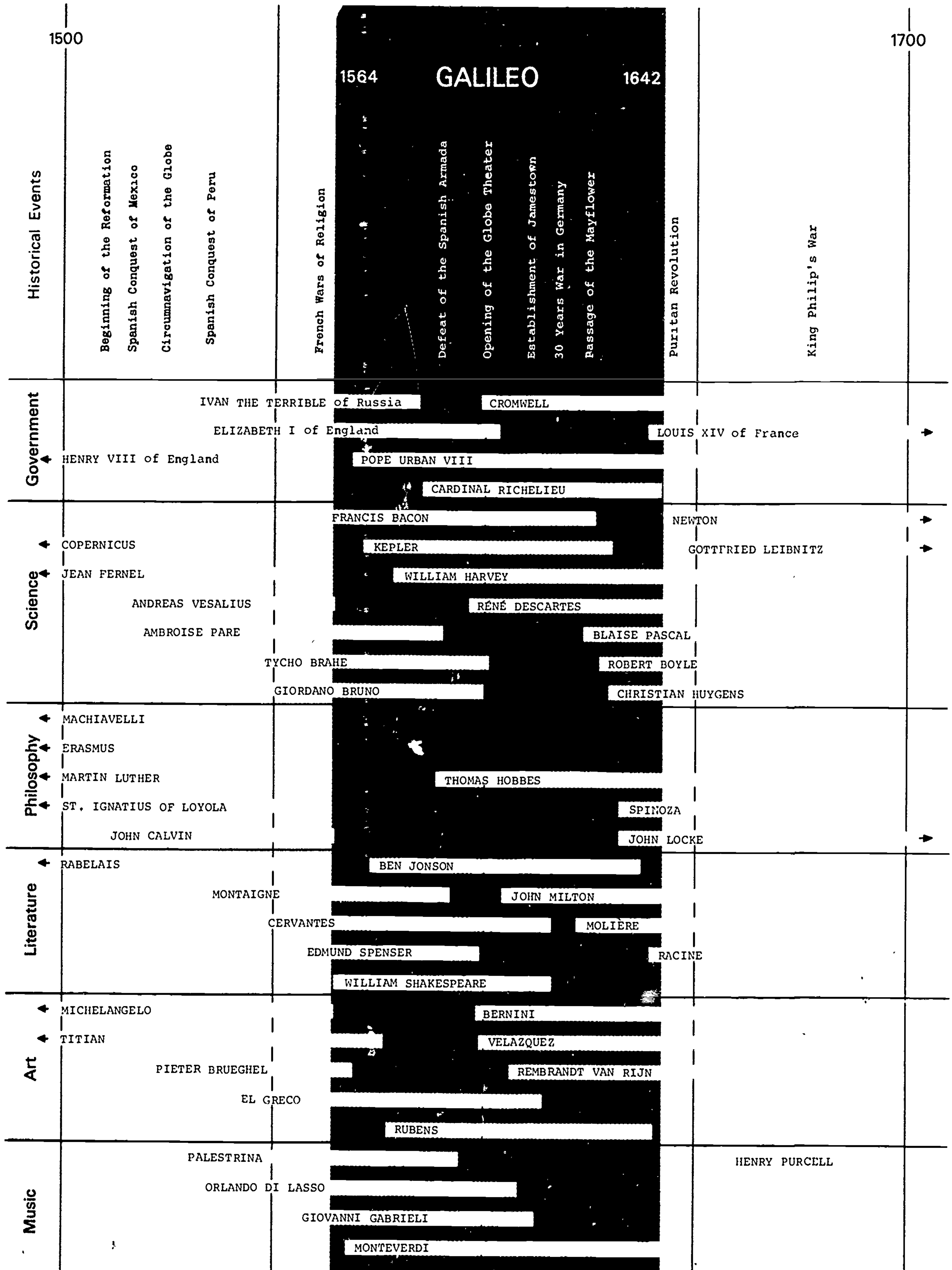
It would not be at all rash to suggest that when, over 19 centuries after Aristotle, Galileo turned his eyes away from all the complicated motions of things in the outside world and fixed them on the curiously artificial motion of a polished brass ball rolling down an inclined plane, his eyes made one of the most important turns in history. And when he succeeded in describing the motion of that ball mathematically he not only paved the way for other men to describe and explain the motions of everything from planets to pebbles but did in fact begin the intellectual revolution which led to what we now call modern science.



2.2 Galileo and his times. The new developments in both physics and astronomy came to focus in the writings of Galileo Galilei. This great scientist was born at Pisa in 1564—the year of Michelangelo's death and Shakespeare's birth. Galileo was the son of a nobleman from Florence and he acquired his father's active interest in poetry, music, and the classics. His scientific inventiveness also began to show itself early. For example, as a young medical student at the University of Pisa, he constructed a simple pendulum-type timing device for the accurate measurement of pulse rates.

Lured from medicine to physical science by reading Euclid and Archimedes, Galileo quickly became known for his unusual ability. At the age of 26, he was appointed Professor of Mathematics at Pisa. There he showed an independence of spirit unmellowed by tact or patience. Soon after his appointment, he began to challenge the opinions of his older

Map of Italy at the time of Galileo.



1500
1700
1564
GALILEO
1642
Historical Events
Beginning of the Reformation
Spanish Conquest of Mexico
Circumnavigation of the Globe
Spanish Conquest of Peru
French Wars of Religion
Defeat of the Spanish Armada
Opening of the Globe Theater
Establishment of Jamestown
30 Years War in Germany
Passage of the Mayflower
Puritan Revolution
King Philip's War
Government
IVAN THE TERRIBLE of Russia
CROMWELL
ELIZABETH I of England
LOUIS XIV of France
HENRY VIII of England
POPE URBAN VIII
CARDINAL RICHELIEU
Science
FRANCIS BACON
NEWTON
COPERNICUS
KEPLER
GOTTFRIED LEIBNITZ
JEAN FERNEL
WILLIAM HARVEY
ANDREAS VESALIUS
RÉNÉ DESCARTES
AMBROISE PARE
BLAISE PASCAL
TYCHO BRAHE
ROBERT BOYLE
GIORDANO BRUNO
CHRISTIAN HUYGENS
Philosophy
MACHIAVELLI
ERASMUS
MARTIN LUTHER
THOMAS HOBBES
ST. IGNATIUS OF LOYOLA
SPINOZA
JOHN CALVIN
JOHN LOCKE
Literature
RABELAIS
BEN JONSON
MONTAIGNE
JOHN MILTON
CERVANTES
MOLIÈRE
EDMUND SPENSER
RACINE
WILLIAM SHAKESPEARE
Art
MICHELANGELO
BERNINI
TITIAN
VELAZQUEZ
PIETER BRUEGHEL
REMBRANDT VAN RIJN
EL GRECO
RUBENS
Music
PALESTRINA
HENRY PURCELL
ORLANDO DI LASSO
GIOVANNI GABRIELI
MONTEVERDI

colleagues, many of whom became his enemies. Indeed, he left Pisa before his term was completed, apparently forced out by financial difficulties and by his enraged opponents. Later, at Padua in the Republic of Venice, he began his work in astronomy. His support of the sun-centered theory of the universe eventually brought him additional enemies, but it also brought him immortal fame. You will read more about this in Unit 2.

Title page of Dialogue on Two Great World Systems (1632).

Drawn back to his native province of Tuscany in 1610 by a generous offer of the Grand Duke, Galileo became the Court Mathematician and Philosopher, a title which he chose himself. From then until his death at 78 in 1642, he produced much of his excellent work. Despite illness, family troubles, occasional brushes with poverty, and quarrels with his enemies, he continued his research, teaching and writing.

Galileo gave us a new mathematical orientation toward the natural world. His philosophy of science had its roots in the ancient Greek tradition of Pythagoras, Plato and Archimedes, but it was in conflict with the qualitative approach characteristic of Aristotle. Unlike most of his predecessors, however, Galileo respected the test of truth provided by quantitative observation and experiment.

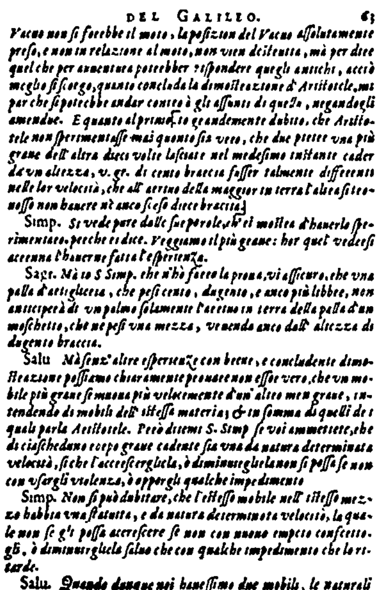
2.3 Galileo's "Two New Sciences." Galileo's early writings on mechanics (the study of the behavior of matter under the influence of forces) were in the tradition of the standard medieval theories of physics. Although he was keenly aware of the short-comings of those theories, his chief interest during his mature years was in astronomy. However, when his important astronomical work, Dialogue on the Two Great World Systems (1632), was condemned by the Roman Catholic Inquisition and he was forbidden to teach the "new" astronomy, Galileo decided to concentrate on mechanics. This work led to his book, Discourses and Mathematical Demonstrations Concerning Two New Sciences Pertaining to Mechanics and Local Motion, usually referred to as the Two New Sciences. The new approach to the science of motion described in the Two New Sciences signaled the beginning of the end not only of the medieval theory of mechanics, but also of the entire Aristotelian cosmology.

DISCORSI
E
DIMOSTRAZIONI
MATEMATICHE,
intorno à due nuoue ſcienze
Attenenti alla
MECANICA & i MOVIMENTI LOCALI,
del Signor
GALILEO GALILEI LINCEO,
Filoſofo e Matematico primario del Sereniſſimo Grand Duca di Toſcana.
Con vna Appendice del centro di grauità d'alcuni Solidi.

IN LEIDA,
Appreſſo gli Elſevirii. M. D. C. XXXVIII.

Title page of Discourses and Mathematical Demonstrations Concerning Two New Sciences Pertaining to Mechanics and Local Motion (1638).

Galileo was old, sick and nearly blind at the time he wrote Two New Sciences, yet his style in it is spritely and delightful. He used the dialogue form to allow a lively conversation between three "speakers": Simplicio, who rep-


DEL GALILEO. 63

Vacuo non si facebbe il moto, la posizion del Vacuo assolutamente presa, e non in relazione al moto, non vien destrutta; mà per dire quel che per avventura potrebber rispondere quegli antichi, acciò meglio si scorga, quanto concluda la dimostrazione d'Aristotele, mi par che si potrebbe andar contro à gli assunti di quello, negandogli amendue. E quanto al primo, io grandemente dubito, che Aristotele non sperimentasse mai quanto sia vero, che due pietre una più grave dell'altra dieci volte lasciate nel medesimo instante cader da un altezza, v. gr. di cento braccia fusser talmente differenti nelle lor velocità, che all'arrivo della maggior in terra l'altra si trovasse non havere nè anco sceso dieci braccia.

Simp. Si vede pure dalle sue parole, ch'ei mostra d'haverlo sperimentato, perche ei dice. Veggiamo il più grave: hor quel vedersi accenna l'haverne fatta l'esperienza.

Sagr. Mà io S. Simp. che n'hò fatto la prova, vi assicuro, che una palla d'artiglieria, che pesi cento, dugento, e anco più libbre, non anticiperà di un palmo solamente l'arrivo in terra della palla d'un moschetto, che ne pesi una mezza, venendo anco dall'altezza di dugento braccia.

Salu. Mà senz'altre esperienze con breve, e concludente dimostrazione possiamo chiaramente provare non esser vero, che un mobile più grave si muova più velocemente d'un'altro men grave, intendendo di mobili dell'istessa materia; & in somma di quelli de i quali parla Aristotele. Però ditemi S. Simp. se voi ammettete, che di ciaschedun corpo grave cadente sia una da natura determinata velocità, sicche l'accrescergliela, ò diminuirgliela non si possa se non con usargli violenza, ò opporgli qualche impedimento

Simp. Non si può dubitare, che l'istesso mobile nell'istesso mezzo habbia una statuita, e da natura determinata velocità, la quale non se gli possa accrescere se non con nuovo impeto conferitogli, ò diminuirgliela salvo che con qualche impedimento che lo ritardi.

Salu. Quando dunque noi havessimo due mobili, le naturali velo-

A page from the original Italian edition of the Two New Sciences, showing Salviati's statement about Aristotle (see translation in text).

resents the Aristotelian view; Salviati, who presents the new views of Galileo; and Sagredo, the uncommitted man of good will and open mind, eager to learn. To no one's surprise, Salviati leads his companions to Galileo's views. Let us listen to Galileo's three speakers as they discuss the problem of free fall:

> Salviati: I greatly doubt that Aristotle ever tested by experiment whether it is true that two stones, one weighing ten times as much as the other, if allowed to fall at the same instant, from a height of, say, 100 cubits, would so differ in speed that when the heavier had reached the ground, the other would not have fallen more than 10 cubits. [A "cubit" is equivalent to about 20 inches.]
>
> Simplicio: His language would seem to indicate that he had tried the experiment, because he says: We see the heavier; now the word see shows that he had made the experiment.
>
> Sagredo: But, I, Simplicio, who have made the test can assure you that a cannon ball weighing one or two hundred pounds, or even more, will not reach the ground by as much as a span ahead of a musket ball weighing only half a pound, provided both are dropped from a height of 200 cubits.

Here, perhaps, one might have expected to find a detailed report on an experiment done by Galileo or one of his colleagues. Instead, Galileo presents us with a "thought experiment"—an analysis of what would happen in an imaginary experiment, in which Galileo ironically uses Aristotle's own method of logical reasoning to attack Aristotle's theory of motion:

> Salviati: But, even without further experiment, it is possible to prove clearly, by means of a short and conclusive argument, that a heavier body does not move more rapidly than a lighter one provided both bodies are of the same material and in short such as those mentioned by Aristotle. But tell me, Simplicio, whether you admit that each falling body acquires a definite speed fixed by nature, a velocity which cannot be increased or diminished except by the use of violence or resistance?
>
> Simplicio: There can be no doubt but that one and the same body moving in a single medium has a fixed velocity which is determined by nature and which cannot be increased except by the addition of impetus or diminished except by some resistance which retards it.
>
> Salviati: If then we take two bodies whose natural speeds are different, it is clear that on uniting the two, the more rapid one will be partly retarded by the slower, and the slower will be somewhat hastened by the swifter. Do you not agree with me in this opinion?
>
> Simplicio: You are unquestionably right.

Salviati: But if this is true, and if a large stone moves with a speed of, say, eight while a smaller moves with a speed of four, then when they are united, the system will move with a speed less than eight; but the two stones when tied together make a stone larger than that which before moved with a speed of eight. Hence the heavier body moves with less speed than the lighter; an effect which is contrary to your supposition. Thus you see how, from your assumption that the heavier body moves more rapidly than the lighter one, I infer that the heavier body moves more slowly.

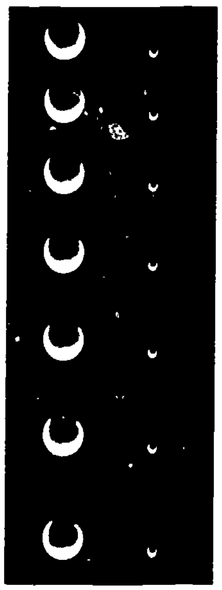
Simplicio: I am all at sea....This is, indeed, quite beyond my comprehension....

See Study Guide 2.5 and 2.14.

As Simplicio retreats in confusion, Salviati presses forward with the argument, showing that it is self-contradictory to assume that an object would fall faster if its weight were increased by a small amount. Simplicio cannot refute Galileo's logic, but on the other hand his own eyes tell him that a heavy object does fall faster than a light object:

Simplicio: Your discussion is really admirable; yet I do not find it easy to believe that a bird-shot falls as swiftly as a cannon ball.

Salviati: Why not say a grain of sand as rapidly as a grindstone? But, Simplicio, I trust you will not follow the example of many others who divert the discussion from its main intent and fasten upon some statement of mine that lacks a hairsbreadth of the truth, and under this hair hide the fault of another that is as big as a ship's cable. Aristotle says that "an iron ball of one hundred pounds falling from a height of 100 cubits reaches the ground before a one-pound ball has fallen a single cubit." I say that they arrive at the same time. You find, on making the experiment, that the larger outstrips the smaller by two fingerbreadths....Now you would not hide behind these two fingers the 99 cubits of Aristotle, nor would you mention my small error and at the same time pass over in silence his very large one.



A stroboscopic photograph of two freely falling balls of unequal weight. The balls were released simultaneously. The time interval between images is 1/30 sec.

This is a clear statement of an important principle: in careful observation of a common natural event the observer's attention may be distracted from a fundamental regularity unless he considers the possibility that small, separately explainable, variations will be associated with the event. Different bodies falling in air from the same height do not reach the ground at exactly the same time. However, the important point is not that the times of arrival are slightly different, but that they are very nearly the same! The failure of the bodies to arrive at exactly the same time is seen to be a minor matter which can be explained by a deeper understanding of motion in free fall. Galileo himself attributed the observed results to the resistance of the air. A few years after Galileo's death, the invention of the air pump allowed others to show that Galileo was right. When a feather and a heavy gold coin are dropped from the same height at the same time inside an evacuated container, they fall at the same rate and strike the bottom of the container at the same instant.

We might say that learning what to ignore has been almost as important in the growth of science as learning what to take into account. In this particular case, Galileo's explanation depended on his being able to imagine how an object would fall if there were no air resistance. This may be easy for us who know of vacuum pumps. But in Galileo's time it was an explanation unlikely to be accepted because of the basic beliefs held by most educated people. For them, as for Aristotle, common sense said that air resistance is always present in nature. Thus, a feather and a coin could never fall at the same rate. Why should one talk about hypothetical motions in a vacuum, when a vacuum does not exist? Physics, said Aristotle and his followers, should describe the real world as we observe it, not some imaginary world which can never be found. Aristotle's physics had dominated Europe since the thirteenth century, not merely because of the authority of the Catholic Church, as is sometimes said, but also because many intelligent scientists were convinced that it offered the most rational method for describing natural phenomena. To overthrow such a firmly established doctrine required much more than writing reasonable arguments or simply dropping heavy and light objects from a tall building, as Galileo is supposed to have done in his legendary experiment on the Leaning Tower of Pisa. It demanded Galileo's unusual combination of mathematical talent, experimental skill, literary style, and tireless campaigning to defeat Aristotle's theories and to get on the path to modern physics.

Another argument against the possibility of a vacuum could be deduced from Aristotle's theory: if the rate of fall is equal to the weight divided by the resistance and the resistance of a vacuum is zero, then the rate of fall of <u>all</u> bodies must be infinite in a vacuum. But that is absurd. Hence, a vacuum is impossible!

2.4 Why study the motion of freely falling bodies? To attack the Aristotelian cosmology, Galileo gathered concepts, methods of calculation, and techniques of measurement in order to describe the motion of objects in a rigorous, mathematical form. Few details of his work were actually new, but together his findings provided the first coherent presentation of the science of motion. He realized that free-fall motion, now seemingly so trite, was the key to the understanding of all motions of all bodies.

By Aristotelian cosmology is meant the whole interlocking set of ideas about the structure of the physical universe and the behavior of all the objects in it. This was briefly and incompletely outlined in Sec. 2.1. Other aspects of it will be presented in Unit 2.

Galileo also provides an example of a superb scientist. He was an investigator whose skill in discovery and eloquence in argument produced a deep and lasting impression on his listeners. His approach to the problems of motion will provide us with an opportunity for discussion of strategies of inquiry that are used in science. We shall see a new mode of scientific reasoning emerge, to become, eventually, an accepted pattern for scientific thought.

These are the reasons why we study in detail Galileo's attack on the problem of free fall. But perhaps Galileo himself should tell us why he studied motion:

> My purpose is to set forth a very new science dealing with a very ancient subject. There is, in nature, perhaps nothing older than motion, concerning which the books written by philosophers are neither few nor small; nevertheless, I have discovered some properties of it that are worth knowing and that have not hitherto been either observed or demonstrated. Some superficial observations have been made, as, for instance, that the natural motion of a heavy falling body is continuously accelerated; but to just what extent this acceleration occurs has not yet been announced....
>
> Other facts, not few in number or less worth knowing I have succeeded in proving; and, what I consider more important, there have been opened up to this vast and most excellent science, of which my work is merely the beginning, ways and means by which other minds more acute than mine will explore its remote corners.

He was wrong in this: more than mere "superficial observations" had been made long before Galileo set to work. For example, Nicolas Oresme and others at the University of Paris had by 1330 discovered the same distance-time relationship for falling bodies that Galileo was to announce with a flourish in the Two New Sciences.

2.5 Galileo chooses a definition of uniform acceleration. In studying the following excerpts from the Two New Sciences, which deal directly with the motion of freely falling bodies, we must be alert to his overall plan. First, Galileo discusses the mathematics of a possible, simple type of motion, namely, motion with uniform acceleration. Then he assumes that this is the type of motion that a heavy body undergoes during free fall. This assumption is his main hypothesis about free fall. Third, he deduces from this hypothesis some predictions that can be tested experimentally. Finally, he shows that these tests do indeed bear out the predictions.

It will help you to have this plan clearly in mind as you progress through the rest of this chapter. As you study each succeeding section, ask yourself whether Galileo is
- presenting a definition
- stating an assumption
- deducing predictions from his hypothesis
- experimentally testing the predictions.

In the first part of Galileo's presentation there is a thorough discussion of motion with uniform speed similar to the one in our Chapter 1. The second part concerns "uniformly accelerated motion":

> We pass now to...naturally accelerated motion, such as that generally experienced by heavy falling bodies....
>
> And first of all it seems desirable to find and explain a definition best fitting natural phenomena. For anyone may invent an arbitrary type of motion and discuss its properties...we have decided to consider the phenomena of bodies falling with an acceleration such as actually occurs in nature and to make this definition of accelerated motion exhibit the essential features of observed accelerated motions.

Finally, in the investigation of naturally accelerated motion we were led, by hand as it were, in following the habit and custom of nature herself, in all her various other processes, to employ only those means which are most common, simple and easy....

This is sometimes known as the rule of parsimony: unless you know otherwise, assume the simplest possible hypothesis to explain natural events.

When, therefore, I observe a stone initially at rest falling from an elevated position and continually acquiring new increments of speed, why should I not believe that such increases take place in a manner which is exceedingly simple and rather obvious to everybody? If now we examine the matter carefully we find no addition or increment more simple than that which repeats itself always in the same manner. This we readily understand when we consider the intimate relationship between time and motion; for just as uniformity of motion is defined by and conceived through equal times and equal spaces (thus we call a motion uniform when equal distances are traversed during equal time-intervals), so also we may, in a similar manner, through equal time-intervals, conceive additions of speed as taking place without complication....

Galileo is saying that just as we have defined uniform speed so that (to use our symbols, not his):

$$v = \frac{\Delta d}{\Delta t},$$

let us also define uniform acceleration so that:

$$a = \frac{\Delta v}{\Delta t}.$$

This is the same definition we used in Chapter 1. Since Galileo always deals with the case of objects falling from rest, this can be written in the form

$$a = \frac{v}{t}.$$

Hence the definition of motion which we are about to discuss may be stated as follows:

A motion is said to be uniformly accelerated, when starting from rest, it acquires during equal time-intervals, equal increments of speed.

Sagredo: Although I can offer no rational objection to this or indeed to any other definition devised by any author whosoever, since all definitions are arbitrary, I may nevertheless without defense be allowed to doubt whether such a definition as the foregoing, established in an abstract manner, corresponds to and describes that kind of accelerated motion which we meet in nature in the case of freely falling bodies....

Here Sagredo, the challenger, questions whether Galileo's arbitrary definition of acceleration actually corresponds to the way real objects fall. Is acceleration, as defined, useful in describing their change of motion? Sagredo tries to divert the conversation:

From these considerations perhaps we can obtain an answer to a question that has been argued by philosophers, namely, what is the cause of the acceleration of the natural motion of heavy bodies....

Salviati, the spokesman of Galileo, sternly turns away from this ancient concern for causes. It is premature, he declares, to ask about the cause of any motion until an accurate description of it exists:

Here Salviati refers to Aristotle's assumption that air propels an object moving through it (Sec. 2.1).

Salviati: The present does not seem to be the proper time to investigate the cause of the acceleration of natural motion concerning which various opinions have been expressed by philosophers, some explaining it by attraction to the center, others by repulsion between the very small parts of the body, while still others attribute it to a certain stress in the surrounding medium which closes in behind the falling body and drives it from one of its positions to another. Now, all these fantasies, and others, too, ought to be examined; but it is not really worth while. At present it is the purpose of our Author merely to investigate and to demonstrate some of the properties of accelerated motion, whatever the cause of this acceleration may be.

Galileo has now introduced two distinct suggestions, which we must take up in turn. 1) "Uniform acceleration" means equal increases in speed Δv in equal times Δt; and 2) things actually fall that way. Let us first look more closely at Galileo's proposed definition.

Is this the only possible way of defining acceleration? Is it obviously right? Not at all! As Galileo goes on to admit, he once believed that in uniform acceleration the speed increased in proportion to the distance traveled, Δd, rather than to the time Δt. In fact, both definitions had been discussed since early in the fourteenth century, and both met Galileo's first command: assume a simple relationship among the physical quantities concerned. Furthermore, both definitions seem to match our commonsense idea of acceleration. For example, when we say that a body is "accelerating," we seem to imply "the farther it goes, the faster it goes," as well as "the longer it keeps moving, the faster it goes." And what, you might ask, is there to choose between these two ways of putting it?

Acceleration could be defined either way. But which definition can be found useful in a description of nature? This is where experimentation is important. Galileo defined uniform acceleration so that change of speed is proportional to elapsed time, and this definition led to fruitful consequences. Other scientists chose to define acceleration so that speed is proportional to distance traversed. Galileo's definition turned out to be the most useful so it was brought into the language of physics.

2.6 Galil cannot test his hypothesis directly. Galileo defined uniform acceleration so that it would match the way he believed freely falling objects behaved. The next task for Galileo was to show that the definition for uniform acceleration ($a = v/t$ = constant) was useful for describing observed facts.

This was not as easy as it seems. Suppose we drop a heavy object from several different heights—say, from windows on different floors of a building. In each case we observe the time of fall t and the speed v just before the object strikes the ground. Unfortunately, it would be very difficult to make direct measurements of the speed v just before striking the ground. Furthermore, the times of fall are smaller (less than 3 sec even from the top of a 10-story building) than Galileo could have measured accurately with the clocks available.

2.7 Looking for logical consequences of Galileo's hypothesis. The inability to make direct measurements to test his hypothesis that v/t is constant did not stop Galileo. He turned to mathematics to derive some other relationship that could be measured with the equipment available to him.

He wanted to answer the question: for an object moving with uniform acceleration what is the relationship between the distance traveled and the time elapsed?

Distance, of course, is easily determined, so Galileo set out to derive an equation for acceleration expressed in terms of distance and time rather than speed and time. We shall derive such an equation by using relationships familiar to us, rather than by following Galileo's derivation exactly. First, we recall the definition of average speed as the distance traversed divided by the elapsed time. In symbols we write

$$v_{av} = \frac{d}{t} .$$

This is a general definition and can be used to compute the average speed for any moving object.

As before,

$v_{initial}$ = initial speed

v_{final} = final speed

v_{av} = average speed.

For the special case of an object moving with uniform acceleration, we can express the average speed in another way—in terms of initial and final speed:

$$v_{av} = \frac{v_{initial} + v_{final}}{2} .$$

If this uniformly accelerating object starts from rest, that is $v_{initial} = 0$, we can write

$$v_{av} = \frac{v_{final}}{2} = \frac{1}{2}v_{final} .$$

See Study Guide 2.6.

In words we would say the average speed of any object starting from rest and accelerating uniformly is one-half the final speed.

We now have two equations which can be applied to the special case of uniformly accelerated motion. Since the average speed is given by both of these equations, we can eliminate v_{av}. Thus,

$$v_{av} = \frac{d}{t} \text{ or } d = v_{av}\, t.$$

So, substituting $\frac{1}{2}v_{final}$ for v_{av} we have

$$d = \frac{1}{2}v_{final}\, t.$$

Does this equation hold for cases of uniform acceleration only?

We now have to take a final step. Somehow we need to get acceleration into the equation and speed out of it. Our starting place was:

$$a = \frac{v_{final}}{t}$$

which, when we solve for v_{final}, becomes

$$v_{final} = at.$$

If we now combine this with

$$d = \tfrac{1}{2}v_{final}\, t$$

we get $$d = \tfrac{1}{2}(at)t$$

or $$d = \tfrac{1}{2}at^2.$$

What is the unwritten text behind this equation?

Galileo's own derivation was somewhat different from this. However, he reached the same conclusion: in uniformly accelerated motion the distance traveled in any time by an object starting from rest is equal to one-half the acceleration times the square of the time. Since we are dealing only with the special case in which acceleration is uniform and $\frac{1}{2}a$ is constant, we can state the conclusion as a proportion: in uniform acceleration the distance traveled is proportional to the square of the time elapsed. For example, if a uniformly accelerating cart moves 3 m in 2 sec, it would move 12 m in 4 sec.

Now let us see where we are with reference to Galileo's problem. Using the three expressions $a = \frac{v_{final}}{t}$, $v_{av} = \frac{v_{final}}{2}$ and $d = v_{av}t$, we found that $d = \frac{1}{2}at^2$. This simple relation, derived from Galileo's definition of acceleration, is the key to an experimental test which he proposed. The relation can be put into a form of more direct interest if we divide it by t^2:

$$\frac{d}{t^2} = \tfrac{1}{2}a\ .$$

Thus a logical result of the original definition of uniform acceleration is: whenever a is constant, the ratio, d/t^2, is constant. Therefore, any motion in which this ratio is constant for different distances and times must be a case of uniform acceleration as defined by Galileo. Of course, it was his hypothesis that freely falling bodies exhibited just such motion.

Galileo's hypothesis restated: for freely falling bodies the ratio d/t^2 is constant. How else could this be worded?

The derived relationship $d/t^2 = \frac{1}{2}a$ has one big advantage over the definition of uniform acceleration: it does not

See Study Guide questions 2.16 and 2.17.

contain the speed v which Galileo had no reli. le way of measuring. Instead, it contains the distance d, which he could measure directly and easily. However, the measurement of the time of fall t remains as difficult as before. Hence, a direct test of his hypothesis still eluded Galileo.

Q1 Why did Galileo use the equation $d = \frac{1}{2}at^2$ rather than $a = \frac{v}{t}$ in testing his hypothesis?

Q2 If you simply combined the two equations $d = vt$ and $a = \frac{v}{t}$ you might expect to get the result $d = at^2$. Why is this wrong?

2.8 Galileo turns to an indirect test. Realizing that it was still impossible to carry out direct quantitative tests with freely falling bodies, Galileo next proposed a related hypothesis which could be tested much more easily. According to Galileo, the truth of his new hypothesis would be established when we find that the inferences from it correspond and agree exactly with experiment.

The new hypothesis is this: if a freely falling body has an acceleration that is constant, then a perfectly round ball rolling down a perfectly smooth inclined plane will also have a constant, though smaller, acceleration. Thus, Galileo claims that if $\frac{d}{t^2}$ is constant for a body falling freely from rest, this ratio will also be constant, although smaller, for a ball released from rest and rolling different distances down an inclined plane.

Here is how Salviati described Galileo's own experimental test:

Note the careful description of the experimental apparatus. Today an experimenter would add to his verbal description any detailed drawings, schematic layouts, or photographs needed to make it possible for any other competent scientist to duplicate the experiment.

> A piece of wooden moulding or scantling, about 12 cubits long, half a cubit wide, and three finger-breadths thick, was taken; on its edge was cut a channel a little more than one finger in breadth; having made this groove very straight, smooth, and polished, and having lined it with parchment, also as smooth and polished as possible, we rolled along it a hard, smooth, and very round bronze ball. Having placed this board in a sloping position, by lifting one end some one or two cubits above the other, we rolled the ball, as I was just saying, along the channel, noting, in a manner presently to be described, the time required to make the descent. We repeated this experiment more than once in order to measure the time with an accuracy such that the deviation between two observations never exceeded one-tenth of a pulse-beat. Having performed this operation and having assured ourselves of its reliability, we now rolled the ball only one-quarter of the length of the channel; and having measured the time of its descent, we found it precisely one-half of the former. Next we tried other distances, comparing the time for the whole length with that for the half, or with that for two-thirds, or three-fourths, or indeed for any fraction; in such experiments,

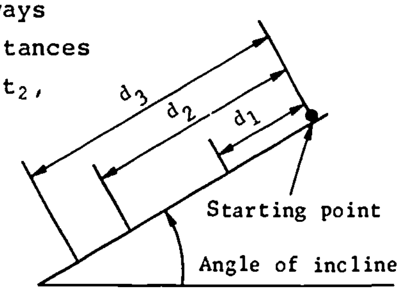
Do you think measurements can actually be made to 1/10-pulse beat? Try it.

repeated a full hundred times, we always found that the spaces traversed were to each other as the squares of the times, and this was true for all inclinations of the...channel along which we rolled the ball....

This picture, painted in 1841 by G. Bezzuoli, reconstructs for us an experiment Galileo is alleged to have made during his time as lecturer at Pisa. To the left and right are men of ill-will: the blasé Prince Giovanni de Medici (Galileo had shown a dredging-machine invented by the prince to be unusable), and Galileo's scientific opponents. These were leading men of the universities, who are bending over a sacrosanct book of Aristotle, where it is written in black and white that, according to the rules of gravity, bodies of unequal weight fall with different speeds. Galileo, the tallest figure left of center in the picture, is surrounded by a group of students.

Galileo has packed a great deal of information into these lines. He describes his procedures and apparatus clearly enough to allow other investigators to repeat the experiment for themselves if they wish; he gives an indication that consistent measurements can be made; and he restates the two experimental results which he believes support his free-fall hypothesis. Let us examine the results carefully.

First, he found that when a ball rolled down an incline at a fixed angle to the horizontal, the ratio of the distance covered to the square of the corresponding time was always the same. For example, if d_1, d_2, and d_3 represent distances from the starting point on the inclined plane, and t_1, t_2, and t_3 the corresponding times, then



$$\frac{d_1}{(t_1)^2} = \frac{d_2}{(t_2)^2} = \frac{d_3}{(t_3)^2}$$

and in general (for a given angle of incline),

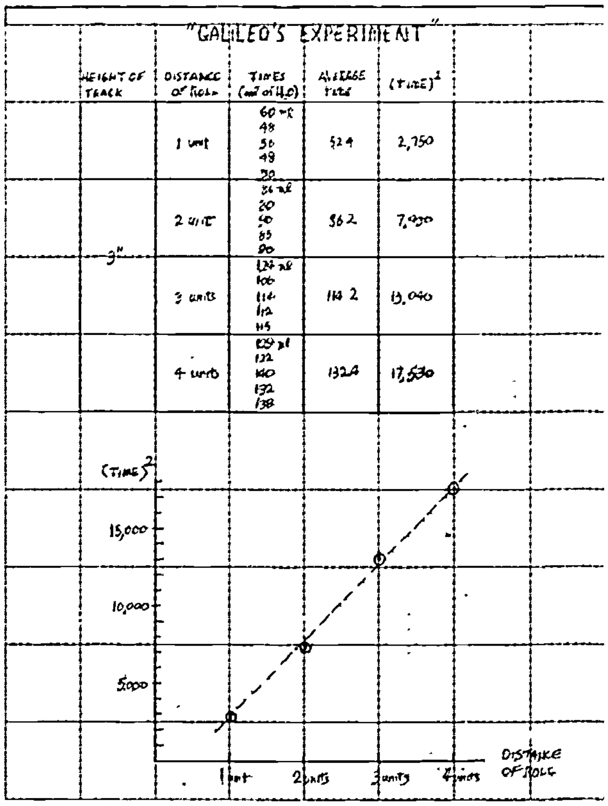
$$\frac{d}{t^2} = \text{constant.}$$

Galileo did not present his experimental data in detail, for that had not yet become the custom. However, his experiment has been repeated by others, and they have obtained results which paralleled his. For example, one experimenter obtained the results shown in Table 2.1. But this is an experiment which you can perform yourself with the help of one or two other students. The students seen conducting this experiment recorded the findings in their notebook shown on the next page.

Table 2.1. Results from an experiment of Thomas Settle in which the angle of inclination was 3° 44' (See Science, 133, 19-23, June 6, 1961).

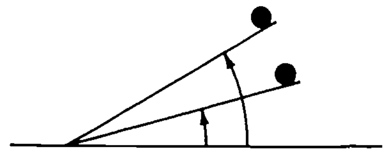
Distance	Time (ml of water)	d/t^2
15 ft	90	.00185
13	84	.00183
10	72	.00192
7	62	.00182
5	52	.00185
3	40	.00187
1	23.5	.00182

Galileo's second experimental finding relates to what happens when the angle of inclination of the plane is changed. He found when the angle changed, the ratio $\frac{d}{t^2}$ also changed, although it was constant for any one angle.



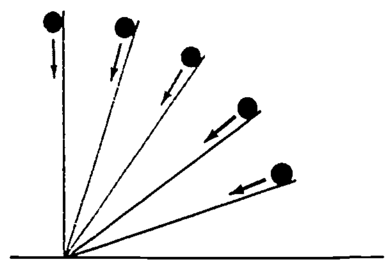
The inclined plane experiment and sample data in a Project Physics class.

This was confirmed by repeating the experiment "a full hundred times" for each of many different angles. After finding that the ratio $\frac{d}{t^2}$ was constant for each angle of inclination for which measurements of t could be carried out conveniently, Galileo was willing to extrapolate.



For each angle, the acceleration is found to be a constant.

He reasoned that the ratio $\frac{d}{t^2}$ is a constant even for larger angles where the motion of the ball is too fast for accurate measurements of t to be made. Further, he reasoned that if the ratio $\frac{d}{t^2}$ is constant when the angle of inclination is 90°, then $\frac{d}{t^2}$ is also a constant for a falling object. Thus, by combining experimentation and reason, Galileo was able to make a convincing argument that for a falling object the ratio $\frac{d}{t^2}$ is a constant.



Spheres rolling down planes of increasingly steep inclination. At 90° the inclined plane situation matches free fall. (Actually, the ball will start slipping long before the angle has become that large.)

Now let us review the steps we have taken. By mathematics we showed that $\frac{d}{t^2}$ = constant is a logical consequence of $\frac{v}{t}$ = constant. In other words, if the statement

$$\frac{v}{t} = \text{constant}$$

is true, then the statement

$$\frac{d}{t^2} = \text{constant}$$

is also true.

Next, Galileo proceeded to prove that $\frac{d}{t^2}$ is a constant for a falling object. By reversing the previous mathematics you can show that if the statement

$$\frac{d}{t^2} = \text{constant}$$

is true, then the statement

$$\frac{v}{t} = \text{constant}$$

must also be true.

But $\frac{v}{t}$ = constant matches Galileo's definition of uniform acceleration, namely

$$a = \frac{v}{t}.$$

Therefore, his hypothesis that falling objects move downward with uniform acceleration appears to be correct.

See Study Guide questions 2.1, 2.2, 2.3, 2.4.

Q3 Galileo's verification of his hypothesis that free fall is uniformly accelerated motion depends on the assumption that

(a) d/t^2 is constant.

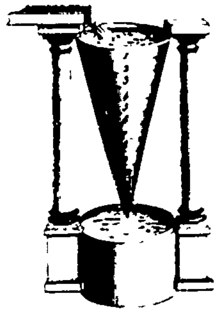
(b) the angle of inclination of the plane does not change.

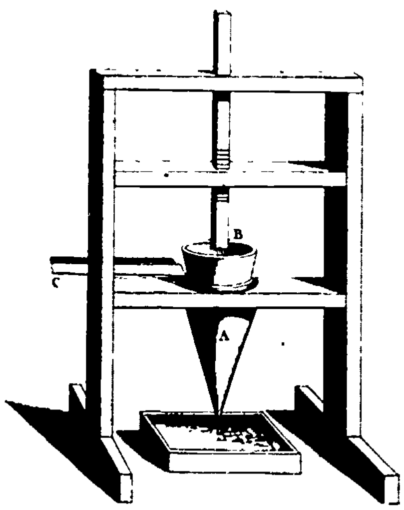
(c) the results for small angles of inclination can be extrapolated to large angles.

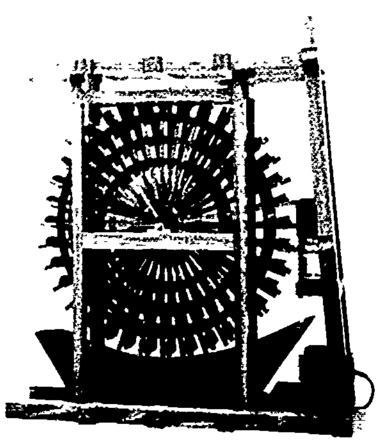
(d) the speed of the ball is constant as it rolls.

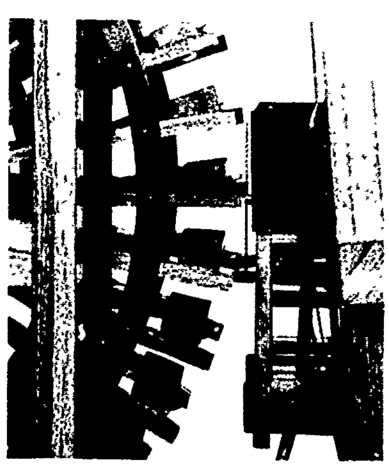
(e) the acceleration of the rolling ball is the same as the acceleration in free fall.

Early Water Clocks.









2.9 <u>How valid was Galileo's procedure</u>? Some doubts arise concerning this whole process of reasoning and experimentation. First, was Galileo's measurement of time accurate enough to establish the constancy of $\frac{d}{t^2}$ even for the earlier case of a slowly rolling object? Galileo tries to reassure possible critics by providing a detailed description of his experimental arrangement (thereby inviting any skeptics to try it for themselves!):

> For the measurement of time, we employed a large vessel of water placed in an elevated position; to the bottom of this vessel was soldered a pipe of small diameter giving a thin jet of water, which we collected in a small cup during the time of each descent, whether for the whole length of the channel or for a part of its length; the water thus collected was weighed on a very accurate balance; the differences and ratios of these weights gave us the differences and ratios of the time intervals, and this with such accuracy that, although the operation was repeated many, many times, there was no appreciable discrepancy in the results.

The water clock described by Galileo was not invented by him. Indeed, there are references to water clocks in China as early as the sixth century B.C., and they were probably used in Babylonia and India even earlier. In Galileo's time, the water clock was the most accurate of the world's time measuring instruments, and it remained so until shortly after his death when the work of Christian Huygens and others resulted in the pendulum clock. Although Galileo's own water clock was not the most precise available at the time, it was, nevertheless, good enough for a convincing verification that $\frac{d}{t^2}$ is constant.

Another reason for questioning Galileo's results is related to the large extrapolation involved. Galileo does not report what angles he used in his experiment. However, as you may have found out from doing a similar experiment, the angles must be kept rather small. Naturally, as the angle increases, the speed of the ball soon becomes so great that it is difficult to measure the times involved. The largest angle reported by Settle in his modern repetition of Galileo's experiment was only 6°. It is unlikely that Galileo worked with much larger angles. This means that Galileo's extrapolation was a large one, perhaps much too large for a cautious person—or for one not already convinced of the truth of Galileo's hypothesis.

Still another reason for questioning Galileo's results is the observation that, as the angle of incline is increased, there comes a point where the ball starts to slide as well as roll. This change in behavior could mean that the same general law does not apply to both cases. Galileo does not answer this objection. It is surprising that he never repeated the experiment with blocks which would slide, rather than roll, down a smooth incline. If he had, he would have found that for both sliding and rolling the ratio $\frac{d}{t^2}$ is a constant although it is a different constant for the two cases.

Q4 The main reason why we might doubt the validity of Galileo's procedure is

(a) his measurement of time was not sufficiently accurate.

(b) he used too large an angle of inclination.

(c) it is not clear that his results apply to the case when the ball can slide as well as roll.

(d) in Galileo's experiment the ball was actually sliding rather than rolling, and therefore his results cannot be extrapolated to the case of free fall.

(e) d/t^2 would not be constant for a sliding object.

2.10 The consequences of Galileo's work on motion. As was pointed out at the end of the previous section, one can not get the correct value for the acceleration of a body in free fall simply by extrapolating the results for larger and larger angles of inclination. In fact Galileo did not even attempt to calculate a numerical value for the acceleration of freely falling bodies. Galileo's purpose could be well served without knowing the value of the acceleration for free fall; it was enough that he showed the acceleration to be constant. This is the first consequence of Galileo's work.

We now know by measurement that the magnitude of the acceleration of gravity, symbol a_g or simply g, is about 9.8 m/sec^2 or 32 ft/sec^2 at the earth's surface (see Study Guide 3.17). The Student Handbook contains five experiments for getting a_g.

Second, if spheres of different weights are allowed to roll down an inclined plane, they have the same acceleration. We do not know how much experimental evidence Galileo himself had for this conclusion. At any rate, later work confirmed his "thought experiment" on the rate of fall of bodies of different weights (Sec. 2.3). The fact that bodies of different weights all fall at the same rate (aside from the understandable effects of air resistance) is a decisive refutation of Aristotle's theory of motion.

Third, Galileo developed a mathematical theory of accelerated motion from which other predictions about motion could be derived. We will mention just one example here, which will turn out to be very useful in Unit 3. Recall that Galileo chose to define acceleration not as the rate of change of speed in a given space, but rather as the rate of change of speed in a given time. He then found by experiment that accelerated bodies in nature actually do experience equal changes of speed in equal times. But one might also ask: if speed does not change by equal amounts in equal distances, is there anything else that does change by equal amounts in equal distances, for a uniformly accelerated motion? The answer is yes: the <u>square</u> of the

Can you derive this equation? (Hint: start from equations for d and v and eliminate t.)

See Study Guide 2.21 and 2.23.

speed changes by equal amounts in equal distances. There is a mathematical equation which expresses this result:

$$v^2 = 2ad$$

In words: if an object starts from rest and moves with uniform acceleration a through a distance d, then the square of its speed will be equal to twice the product of its acceleration and the distance it has moved. We shall see the importance of $v^2 = 2ad$ in Unit 3.

These consequences of Galileo's work, important as they are to modern physics, would scarcely have been enough to bring about a revolution in science by themselves. No sensible person in the seventeenth century would have given up his belief in the Aristotelian cosmology simply because its predictions had been refuted in the case of falling bodies. The significance of Galileo's work is that it prepared the way for the development of a new kind of physics, and indeed a new cosmology.

The more vexing scientific problem during Galileo's lifetime was not the motion of accelerated bodies, but the structure of the universe. For example, is the earth or the sun the center of the universe? Galileo supported the theory that the earth and other planets revolve around the sun. To accept such a theory meant, ultimately, to reject the Aristotelian cosmology; but in order to do this a physical theory of the motion of the earth would have to be developed. Galileo's theory of motion turned out to be just what was needed for this purpose, but only after it had been combined with further assumptions about the relation between forces and motion by the English scientist Isaac Newton. We shall return to the story of this revolution in science in Unit 2.

There is another significant aspect of Galileo's work on motion: it led to a new way of doing scientific research. The heart of this approach is the cycle, repeated as often as necessary: general observation → hypothesis → mathematical analysis → experimental test → modification of hypothesis as necessary in light of test, and so forth. But while the steps in the mathematical analysis are determined by "cold logic," this is not the case for the other elements. Thus a variety of paths can lead to the hypothesis in the first place: an inspired hunch based on general knowledge of the experimental facts, a desire for simple and pleasing foundations, a change of a previous hypothesis that failed. Moreover, there are no general rules about how well the experimental data have to agree with the theoretical predictions. In some areas of science, a theory is expected to be accurate to better than one 1/1000th of a percent; in other areas, scientists would be delighted to find a theory that could make predictions with as little as 50 percent error.

The process of proposing and testing hypotheses, so skillfully demonstrated by Galileo in the seventeenth century, is widely used by scientists today. It is perhaps the most significant thing that distinguishes modern science from ancient and medieval science. The method is used not out of respect for Galileo as a towering figure in the history of science, but because it works so well so much of the time.

Galileo himself was aware of the value of both the results and the methods of his pioneering work. He concluded his treatment of accelerated motion by putting the following words into the mouths of the commentators on his book:

> Sagredo: I think we may concede to our Academician, without flattery, his claim that in the principle laid down in this treatise he has established a new science dealing with a very old subject. Observing with what ease and clearness he deduces from a single principle the proofs of so many theorems, I wonder not a little how such a question escaped the attention of Archimedes, Appolonius, Euclid and so many other mathematicians and illustrious philosophers, especially since so many ponderous tomes have been devoted to the subject of motion.

The "Academician" is the author of the treatise being discussed in the dialogues—that is, Galileo himself.

> Salviati: ...we may say the door is now opened, for the first time, to a new method fraught with numerous and wonderful results which in future years will command the attention of other minds.

> Sagredo: I really believe that...the principles which ar et forth in this little treatise will, when taken up by speculative minds, lead to another more remarkable result; and it is to be believed that it will be so on account of the nobility of the subject, which is superior to any other in nature.

> During this long and laborious day, I have enjoyed these simple theorems more than their proofs, many of which, for their complete comprehension, would require more than an hour each; this study, if you will be good enough to leave the book in my hands, is one which I mean to take up at my leisure after we have read the remaining portion which deals with the motion of projectiles; and this if agreeable to you we shall take up tomorrow.

> Salviati: I shall not fail to be with you.

Projectile motion will be taken up in Chapter 4.

Q5 Which of the following was not a consequence of Galileo's work on motion?

(a) The correct numerical value of the acceleration in free fall was obtained by extrapolating the results for larger and larger angles of inclination.

(b) If an object starts from rest and moves with uniform acceleration a through a distance d, then the square of its speed will be proportional to a and also proportional to d.

(c) Bodies moving on a smooth inclined plane are uniformly accelerated (according to Galileo's definition of acceleration).

2.1 List the steps by which Galileo progressed from his first definition of uniformly accelerated motion to his final confirmation that this definition is useful in describing the motion of a freely falling body. Identify each step as a hypothesis, deduction, observation, or computation, etc. What limitations and idealizations appear in the argument?

2.2 Which of the following statements best summarizes the work of Galileo on free fall? (Be prepared to defend your choice.) Galileo:

a) proved that all objects fall at exactly the same speed regardless of their weight.
b) proved that for any one freely falling object, the ratio: $\frac{d}{t^2}$ is constant for any distance.
c) demonstrated conclusively that an object rolling down a smooth incline accelerates in the same way as (although more slowly than) the same object falling freely.
d) used logic and experimentation to verify indirectly his assertion that the speed of a freely falling object at any point depends only upon, and is proportional to, the time elapsed.
e) made it clear that until a vacuum could be produced, it would not be possible to settle the free-fall question once and for all.

2.3 Write a short statement (not more than two or three sentences) summarizing Galileo's work on free fall better than any of those in 2.2 above.

2.4 As Director of Research in your class, you receive the following research proposals from physics students wishing to improve upon Galileo's free-fall experiment. Would you recommend support for any of them? If you reject a proposal, you should make it clear why you do so.

a) Historians believe that Galileo never dropped objects from the Leaning Tower of Pisa. Too bad! Such an experiment is more direct and more fun than inclined plane experiments, and of course, now that accurate stopwatches are available, it can be carried out much better than in Galileo's time. The experiment involves dropping, one by one, different size spheres made of copper, steel, and glass from the top of the Leaning Tower and finding how long it takes each one to reach the ground. Knowing d (the height of the tower) and time of fall t, I will substitute in the equation $d = \frac{1}{2}at^2$ to see if the acceleration a has the same value for each sphere.

b) A shotput will be dropped from the roof of a 4-story building. As the shotput falls, it passes a window at each story. At each window there will be a student who starts his stopwatch upon hearing a signal that the shot has been released, and stops the watch as the shot passes his window. Also, each student records the speed of the shot. From his own data, each student will compute the ratio v/t. All four students should obtain the same numerical value of the ratio.

c) Galileo's inclined planes "dilute" motion all right, but the trouble is that there is no reason to suppose that a ball rolling down a board is behaving like a ball falling straight downward. A better way to accomplish this is to use light, fluffy, cotton balls. These will not drop as rapidly as metal spheres, and therefore it would be possible to measure the time of the fall t for different distances d. The ratio d/t^2 could be determined for different distances to see if it remained constant. The compactness of the cotton ball could then be changed to see if a different value was obtained for the ratio.

2.5 Consider Aristotle's statement "A given weight moves [falls] a given distance in a given time; a weight which is as great and more moves the same distance in less time, the times being in inverse proportion to the weights. For instance, if one weight is twice another, it will take half as long over a given movement." [<u>De Caelo</u>]

Indicate what Simplicio and Salviati each would predict for the rest of the falling motion in these cases:

a) A two-pound rock falls from a cliff and, while dropping, breaks into two equal pieces.
b) A hundred-pound rock is dropped at the same time as one hundred one-pound pieces of the same type of rock.
c) A hundred one-pound pieces of rock, falling from a height, drop into a loosely held sack which pulls loose and falls.

All the rocks are in the sack and continue falling while contained by the sack.

2.6 A good deal of work preceded that of Galileo on the topic of motion. In the period (1280-1340) mathematicians at Merton College, Oxford, carefully considered quantities that change with the passage of time. One result that had profound influence was a general theorem known as the "Merton Theorem" or "Mean Speed Rule."

This theorem might be restated in our language and applied to uniform acceleration as follows: the distance an object goes during some time while its speed is changing uniformly is the same distance it would go if it went at the average speed the whole time.

Using a graph, and techniques of algebra and geometry, construct a proof of the "Merton Rule."

2.7 In the <u>Two New Sciences</u> Galileo states, "...for so far as I know, no one has yet pointed out that the distances traversed, during equal interval of time, by a body falling from rest, stand to one another in the same ratio as the odd numbers beginning with unity (namely 1:3:5:7...)...."

The area beneath a speed-time graph represents the distance traveled during some time interval. Using that idea, give a proof that the distances an object falls in successive equal time intervals will be in such a ratio.

2.8 Indicate whether the following statements are true or false when applied to the strobe photo at the right:

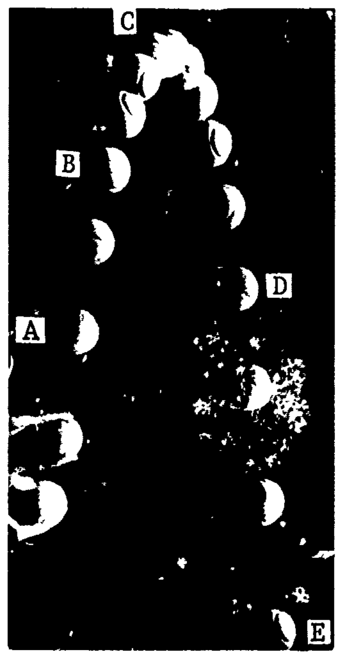
a) The speed of the ball is greater at the bottom than at the top.
b) The direction of the acceleration is vertically downward.
c) This could be a freely falling object.
d) This could be a ball thrown straight upward.

2.9 Apply the same statements to the photo at the right, once again indicating whether each statement is true or false.

2.10 These last two questions raise the issue of direction. The photograph in the figure below is of a ball thrown upward, yet its acceleration is downward. The acceleration due to gravity may appear as the slowing down of an upward moving object, or as the speeding up of a downward moving one. To keep these matters straight, a <u>plus and minus sign convention</u> is adopted. Such a convention is merely an arbitrary but consistent set of rules.

The main rule we adopt is: <u>up</u> is the <u>positive</u> direction. It follows that the acceleration due to free fall g always takes the negative sign; distances above the point of release are positive, those below it negative; and the speed of an object moving upward is positive, downward negative.

The figure below is a photo of the path that a ball might take if you threw it up and then let it fall to the ground rather than catching it when it reached your hand again. To assure yourself that you understand the sign convention stated above, complete the table below.

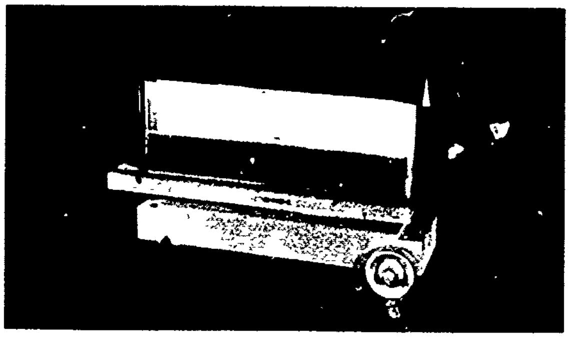


Stroboscopic photograph of a ball thrown into the air.

position	d	v	a
A	+	+	
B			
C			
D			
E			

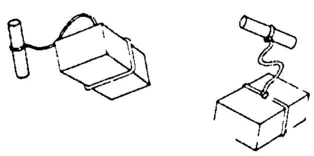
2.11 Draw a set of points (as in a strobe photo) to show the successive positions of an object that had a positive acceleration upward. Can you think of any way to produce such an event physically?

2.12 The instrument shown below on a cart is called a liquid surface accelerometer. Whenever the accelerometer experiences an acceleration in a direction parallel to its long dimension, the surface of the liquid tilts in the direction of the acceleration. Design a demonstration in which acceleration remains constant but speed and direction change.



2.13 Drop sheets of paper with various degrees of "crumpling." Can you crumple a sheet of paper tight enough that it will fall at the same rate as a tennis ball?

2.14 Tie two objects (of greatly different weight) together with a piece of string. Drop the combination with different orientations of objects. Watch the string. In a few sentences summarize your results.



2.15 In these first two chapters we have been concerned with motion in a straight line. We have dealt with distance, time, speed and acceleration, and with the relationships between them. Surprisingly, most of the results of our discussion can be summarized in the three equations listed below.

$$v_{av} = \frac{\Delta d}{\Delta t} \qquad a_{av} = \frac{\Delta v}{\Delta t} \qquad d = \tfrac{1}{2}at^2$$

The last of these equations applies only to those cases where the acceleration is constant. Because these three equations are so useful, they are worth remembering.

a) State each of the three equations in words.
b) Which of the equations can be applied only to objects starting from rest?
c) Make up a simple problem to demonstrate the use of each equation. For example: How long will it take a jet plane to travel 3200 miles if it averages 400 mi/hr? Also work out the solution just to be sure the problem can be solved.

2.16 Memorizing equations will not save you from having to think your way through a problem. You must decide if, when and how to use equations. This means analyzing the problem to make certain you understand what information is given and what is to be found. Test yourself on the following problem. Assume that the acceleration due to gravity is 10 m/sec^2.

Problem: A stone is dropped from rest from the top of a high cliff.

a) How far has it fallen after 1 second?
b) What is the stone's speed after 1 second of fall?
c) How far does the stone fall during the second second? (That is, from the end of the first second to the end of the second second.)

2.17 Think you have it now? Test yourself once more. If you have no trouble with this, you may wish to try problem 2.18, 2.19, or 2.20.

Problem: An object is thrown straight upward with an initial velocity of 20 m/sec.

a) What is its speed after 1.0 sec?
b) How far did it go in this first second?
c) How long did the object take to reach its maximum height?
d) How high is this maximum height?
e) What is its final speed just before impact?

2.18 A batter hits a pop fly that travels straight upwards. The ball leaves his bat with an initial speed of 40 m/sec.

a) What is the speed of the ball at the end of 2 seconds?
b) What is its speed at the end of 6 seconds?
c) When does the ball reach its highest point?
d) How high is this highest point?
e) What is the speed of the ball at the end of 10 seconds? (Graph this series of speeds.)
f) What is its speed when caught by the catcher?

2.19 A ball starts up an inclined plane with a speed of 4 m/sec, and comes to a halt after 2 seconds.

a) What acceleration does the ball experience?
b) What is the average speed of the ball during this interval?
c) What is the ball's speed after 1 second?
d) How far up the slope will the ball travel?
e) What will be the speed of the ball 3 seconds after starting up the slope?
f) What is the total time for a round trip to the top and back to the start?

2.20 Lt. Col. John L. Stapp achieved a speed of 632 mph (284 m/sec) in an experimental rocket sled at the Holloman Air Base Development Center, Alamogordo, New Mexico, on March 19, 1954. Running on rails and propelled by nine rockets, the sled reached its top speed within 5 seconds. Stapp survived a maximum acceleration of 22 g's in slowing to rest during a time interval of 1½ seconds. (22 g's means $22 \times a_g$.)

a) Find the average acceleration in reaching maximum speed.
b) How far did the sled travel before attaining maximum speed?
c) Find the average acceleration while stopping.

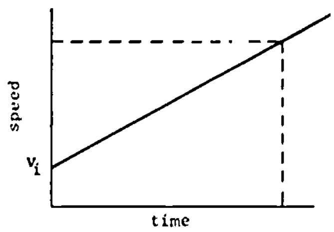
2.21 Sometimes it is helpful to have a special equation relating certain variables. For example, initial and final speed, distance, and acceleration are related by the equation

$$v_f^2 = v_i^2 + 2ad.$$

Try to derive this equation from some others you are familiar with.

2.22 Use the graph below, and the idea that the area under a curve in a speed-time graph gives a value for the distance traveled, to derive the equation

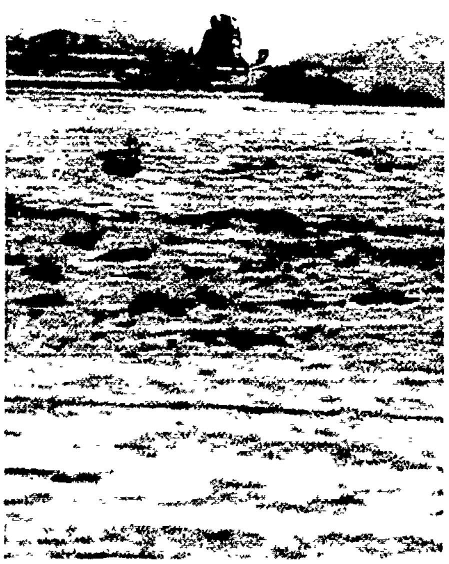
$$d = v_i t + \tfrac{1}{2}at^2.$$



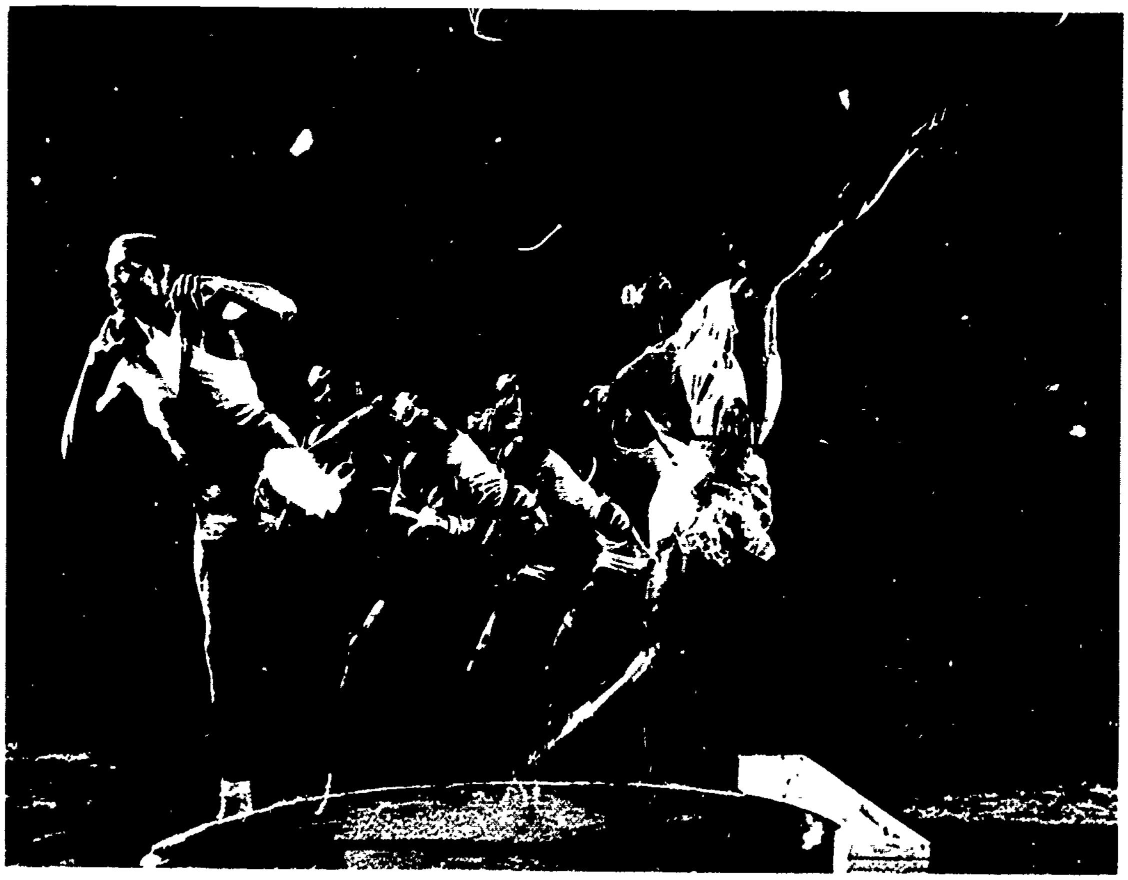
2.23 A student on the planet Arret in another solar system dropped an object in order to determine the acceleration due to gravity. The following data are recorded (in local units):

Time (in surgs)	Distance (in welfs)
0.0	0.00
0.5	0.54
1.0	2.15
1.5	4.84
2.0	8.60
2.2	10.41
2.4	12.39
2.6	14.54
2.8	16.86
3.0	19.33

a) What is the acceleration due to gravity on the planet Arret, expressed in welfs/surg^2?
b) A visitor from Earth finds that one welf is equal to about 6.33 cm and that one surg is equivalent to 0.167 sec. What would this tell us about Arret?



Chapter 3 The Birth of Dynamics-Newton Explains Motion



3.1 The concepts of mass and force. Galileo investigated many topics in mechanics with insight, ingenuity and gusto. The most valuable part of that work dealt with special types of motion, such as free fall. In a clear and consistent way, he developed useful schemes for describing how objects move. Kinematics is the study of how objects move.

When Isaac Newton began his studies of motion in the second half of the seventeenth century, Galileo's earlier insistence that "the present does not seem to be the proper time to investigate the cause of the acceleration of natural motion...." was no longer valid. Indeed, largely because Galileo had been so effective in describing motion, Newton could turn his attention to dynamics; that is, to the question of why objects move the way they do.

How does dynamics differ from kinematics? As we have seen in the two earlier chapters, kinematics deals with the description of motion. Dynamics goes beyond kinematics by taking into account the cause of the motion. For example, in describing the motion of a stone dropped from a cliff, we might include the height from which the stone is dropped and the time the stone remains in its fall. With this information we could compute the stone's average speed and its acceleration. But, when we have completed our description of the stone's motion, we are still not satisfied. Why, we might ask, does the stone accelerate rather than fall with a constant speed? Why does it accelerate uniformly? To answer these questions, we must add to our arsenal of concepts those of force and mass; and in answering, we are doing dynamics.

Kinematic Concepts

position
time
speed
acceleration

Dynamic Concepts

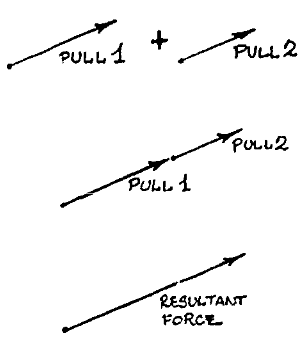
mass
force
momentum (Ch. 9)
kinetic energy (Ch. 10)

Fortunately, the concepts of force and mass are not exactly new. Our common sense idea of force is closely linked with our own muscular activity. We know a sustained effort is required to lift and support a heavy stone. When we push a lawnmower, row a boat, draw a bow, or pull a sled, our muscles let us know we are exerting a force. Perhaps you notice how naturally force and motion and muscular activity are united in our minds. In fact, when you think of moving or changing the motion of an object (e.g., hitting a baseball), you naturally think of the muscular sensation of exerting a force.

The idea of mass is a little more subtle. You have used the word mass, but common sense alone does not lead to a useful definition. Certainly it does not have to do only with size—a brick is more massive than a beachball. Think of a grapefruit and a shot put. Which has a greater mass? The shot put, you say. But why? Because it is heavier? Is mass merely a synonym for weight? No, because if an astronaut smuggled the shot put and the grapefruit aboard his space capsule, then once the vehicle is in a region where the gravitational pull is no longer felt, it will still be much more difficult to accelerate the shot put in throwing it forward (as on p. 64) than would be a grapefruit. Even in the absence of weight, mass remains, and one is felt to be more massive than the other. Probably we can agree that mass is a measure of the quantity of matter in any object; but even this does not solve our difficulty. The question still remaining is, "What is meant by 'quantity of matter'?"

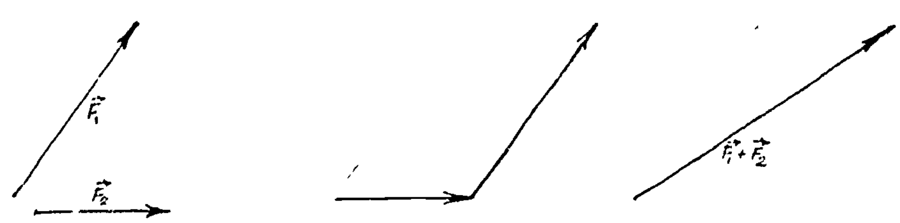
Newton did not "discover" the concepts of force and mass. What he did was this: first, he recognized that these concepts were basic for an understanding of motion and second, he clarified these concepts and defined them in a way that made them extremely useful. In the mind of Newton, the concepts of force and mass became more than fuzzy, qualitative notions—he found a way to attach numbers to them. This may not sound like much. But, by the end of this chapter, perhaps you will agree that Newton's contribution was indeed extraordinary.

3.2 About vectors. Force is a vector. If you are asked to push a piece of furniture from one part of a roc… to another, you size up the situation as follows. First, your experience suggests to you the magnitude of the force required. A force of greater magnitude is required to move a piano than to move a foot stool. Second, you determine the direction in which the force must be applied to make the desired move. Obviously, both the magnitude and the direction of the force are important.



A vector is represented by an arrow-headed line segment whose length is drawn proportional to the magnitude of the vector and whose direction is the same as the vector.

We cannot define a vector until we understand how two vectors are added together. If two forces of equal magnitude, one directed due east and the other directed due north, are applied to a resting object free to move, it will take off in the northeast direction—the direction of the resultant force. The resultant force is the sum of the individual forces. The resultant force is found by application of the rule for vector addition—the parallelogram law. The parallelogram law is illustrated below.



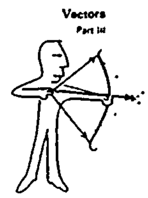
Now we can define a vector. Something which has both magnitude and direction, and which adds by the parallelogram law, is a vector. A surprising variety of things have both magnitude and direction and add together according to the parallelogram law. For example, displacement, velocity and acceleration are vector quantities. Concepts such as volume, distance, or speed do not require specifying a direction in space, and are called scalar quantities.

A vector is labeled by a letter with an arrow over it; for example, $\vec{F}$, $\vec{a}$, or $\vec{v}$.

In Sec. 1.8 we hedged a bit on our definition of acceleration. There we defined acceleration as the rate of change in speed. That is correct, but it is incomplete. Now we want to consider the direction of motion as well. We shall define acceleration as the rate of change of velocity where velocity is a vector having both magnitude and direction. In symbols,

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$$

where $\Delta\vec{v}$ is the change in velocity. Velocity can change in two ways: by changing its magnitude (speed) and by changing its direction. In other words, an object is accelerating when it speeds up, slows down, or changes direction.



We will use vectors frequently. To learn more about them, ask your teacher for the Project Physics program Vectors.

Q1 What is the difference between speed and velocity?

Q2 An object is moving with a velocity of 10 m/sec due north. Five seconds later, it is moving with a velocity of 10 m/sec due east.

(a) What is the change in the velocity $\Delta\vec{v}$?

(b) What is the average acceleration $\Delta\vec{v}/\Delta t$?

3.3 Explanation and the laws of motion. So far in our study of kinematics, we have encountered four situations: an object might

a) remain at rest

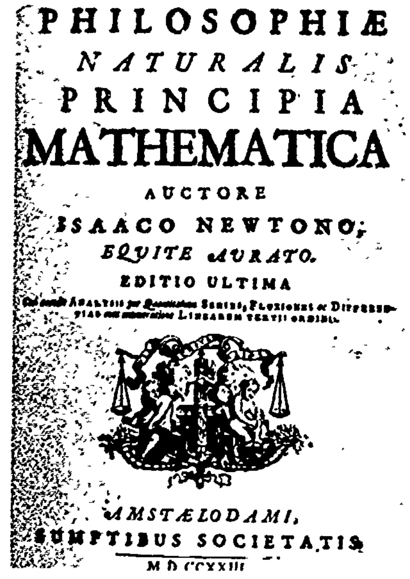
b) move uniformly in a straight line

c) speed up, and

d) slow down.

Because the last two of these are examples of acceleration the list could really be reduced to 1) rest, 2) uniform rectilinear motion, and 3) acceleration. These are the phenomena that we shall first try to explain.

The words "explain" or "cause" have to be used with care. To the physicist, an event is "explained" when he can demonstrate that the event is a logical consequence of a law he assumes to be true. In other words, a physicist,


PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA
AUCTORE
ISAACO NEWTONO,
EQUITE AURATO.
EDITIO ULTIMA
AMSTÆLODAMI,
SUMPTIBUS SOCIETATIS
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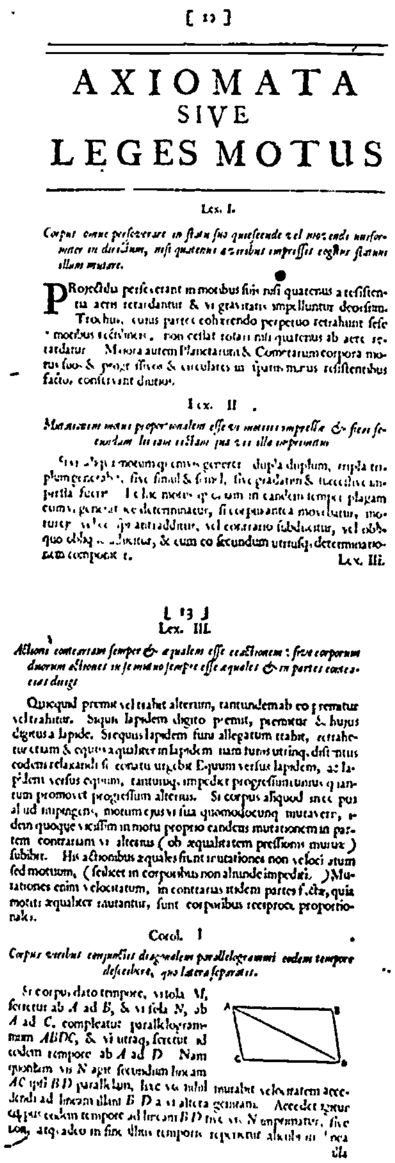

AXIOMATA
SIVE
LEGES MOTUS
Lex. I.
Lex. II.
Lex. III.
Corol. I

with faith in a law, "explains" an observation by showing that it is consistent with the law. In a sense, the physicist's job is to show that the infinity of separate, different-looking occurrences are merely different manifestations of a few general rules on which the world is built.

To explain rest, uniform motion and acceleration, we must be able to answer such questions as: why does a vase placed on a table remain stationary? If a dry ice puck resting on a smooth, level surface is given a brief push, why does it move with uniform speed in a straight line, rather than slow down noticeably or curve to the right or left?

Answers to these (and almost all) specific questions about motion are contained either directly or indirectly in the three general laws of motion formulated by Newton. These laws appear in his famous book, Philosophiae Naturalis Principia Mathematica, which is usually referred to simply as The Principia. We shall examine Newton's three laws of motion one by one. If you are curious about these laws—and if your Latin is fairly good—you might try translating them from the original (shown in the margin). A modernized version of Newton's laws, in English, is in Study Guide 3.1.

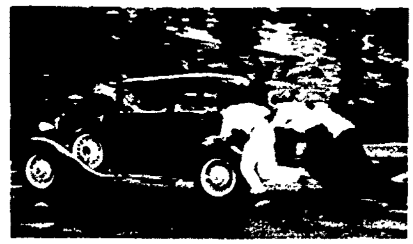
Before looking at Newton's contribution let us first find out how some other scientists of Newton's time might have responded to some of these questions.

3.4 The Aristotelian explanation of motion. The idea of force played a central role in the dynamics of Aristotle, twenty centuries before Newton. You will recall from Chapter 2 that there were two types of motion—"natural" motion and "violent" motion. For example, a falling stone is n natural motion. A stone being steadily lifted is in violent motion. To maintain this uniform violent motion, a force must be continuously applied. A person lifting a large stone is keenly aware that a continual force is required as he strains to hoist the stone higher.

Let us explore the idea of violent or unnatural motion a little further, for as we shall see, there were difficulties. To understand these difficulties, let us take a specific example—an arrow flying through the air. Aristotle had generalized the common-sense notion that an object cannot undergo violent motion without a mover, or something pushing on it. Thus, an arrow flying through the air must be continually propelled by a force. Further, if the propelling force is removed, the arrow should immediately stop its flight and fall directly to the ground. But how can this be? Does the arrow fall to the ground as soon as it loses direct contact with the bow string? The archer —and certainly the victim—are aware that it does not.

What then is the force that propels the arrow? This force was accounted for by an ingenious suggestion: the motion of the arrow was maintained by the air itself! A commotion is set up in the air by the initial movement of the arrow; that is, as the arrow starts to move the air is compressed and pushed aside. The rush of air to fill the space being vacated by the arrow (remember that according to Aristotle a vacuum is impossible) maintains it in its flight.

To an Aristotelian, a force is necessary to sustain uniform motion. The explanation of uniform motion is reduced to identifying the origin of the force. And that is not always easy.



Keeping an object in motion at uniform (constant) velocity.

Of course, Aristotle's followers had other problems. For example, a falling acorn does not move with uniform speed—it accelerates. How is acceleration explained? Aristotelians thought the speeding up of a falling object was associated with its approaching arrival at its natural place, or home, the earth. In other words, a falling object is like the tired old horse that perks up and starts to gallop as it approaches the barn. Galileo's Aristotelian contemporaries offered a more scienti᾿ic-sounding but equally false expl:nation for the accele. ıtion of falling bodies. They claimed that when an object falls, the weight of the air above it increases while the column of air below it decreases, thus offering less resistance to its fall.

When a falling acorn finally reaches the ground, as close to the center of the earth as it can get, it stops. And there, in its natural place, it remains. Rest, the natural state of the acorn, requires no explanation. You see, the three phenomena—rest, uniform motion, and acceleration—could be explained by an Aristotelian. Now, let us examine the alternative explanation that our present understanding offers.

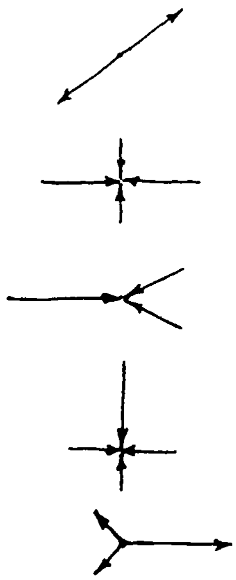
Study Guide 3.2

Q3 According to Aristotle, a ________ is necessary to maintain motion.

Q4 Can you come up with an Aristotelian explanation of a dry-ice puck's uniform motion across a table top?

3.5 Forces in equilibrium.

Forces make things move—they also hold things still. The barrel supporting a circus elephant and the cable supporting the main span of the Golden Gate Bridge are both under the influence of mighty forces, yet they remain at rest. Apparently, more is required to initiate motion than the mere application of a force.



In which cases are the forces balanced?

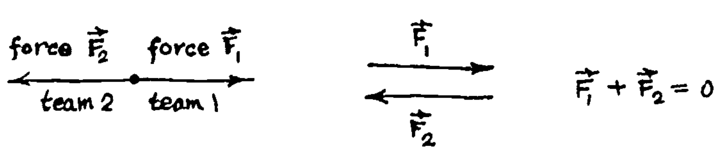
Of course, this may not be surprising to you. We have all seen children quarrelling over the same toy. If each child pulls determinedly in his own direction, the toy goes nowhere. On the other hand, if two of the children cooperate and pull together against the third, then the tide of battle shifts.

Likewise, in a tug-of-war between two teams, there are large forces exerted at each end, but the rope may not budge an inch. You might say it is all a matter of balance. If the team pulling in one direction exerts a force equal to that of the team pulling in the other direction, the forces acting on the rope are balanced and the rope does not move. The physicist would say that the rope is in equilibrium under the forces acting on it.

For reasons explained in the next section, we shall have to make a correction, and add "or in uniform rectilinear motion" wherever the word "rest" appears in this section.

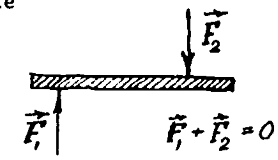
The vector nature of force suggests a graphical representation of the tug-of-war or the toy-pulling episode. When we draw the lengths of the vectors representing the forces acting on the rope or toy proportional to the magnitudes of the forces, we discover a surprising result. We can predict whether or not the rope or the toy will remain at rest! In fact, if we know the forces acting on any object, we can generally predict whether an object at rest will remain at rest.

It is as simple as this: if the vectors representing the forces acting on an object at rest add up to zero, the object is in equilibrium under these forces and will remain at rest. To return to our tug-of-war, let us assume the forces are known and are accurately represented by the vectors drawn below. They are balanced; that is, the net force is zero.

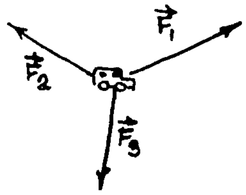


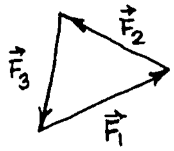
This same procedure can be applied to the toy. Again, the known forces are represented by vectors and are drawn below. Is the toy in equilibrium under the forces? Yes, if the vectors add up to zero. Let's see.

We are defining equilibrium without worrying whether the object will rotate. For example



The sum of the forces on the plank in the diagram is zero, but it is obvious that the plank will rotate.





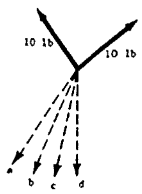
$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

Yes, indeed, the truck is in equilibrium. To obtain the answer, we merely apply the rule of vector addition. A ruler and protractor are, of course, handy tools of the trade.

We can now summarize our understanding of the state of rest as follows: if the sum of all forces acting on an object at rest is zero, the object will remain at rest. We regard rest as a condition or state in which forces are balanced.

Study Guide 3.3

Q5 Which arrow (a,b,c, or d) indicates the direction and magnitude of the force needed to balance the two 10-pound forces indicated in the diagram?



3.6 Newton's first law of motion. Were you surprised when you first pushed a dry-ice puck or some other frictionless device? Remember how it glided along after just the slightest nudge? Remember how it showed no signs of slowing down? Or speeding up? We were surprised, probably because the puck failed to live up to our everyday Aristotelian expectations.

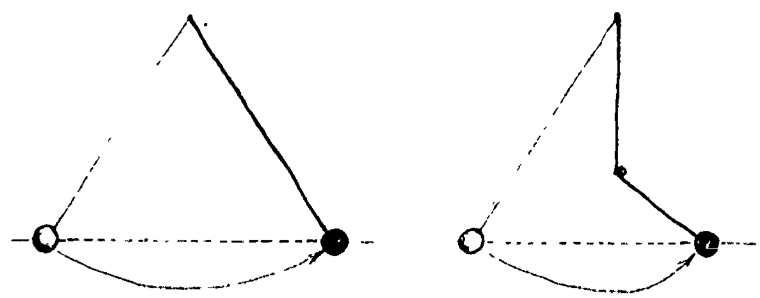
Yet the puck was behaving quite naturally indeed. If the retarding forces of friction were absent, a gentle push would cause tables and chairs to glide across the floor just like a dry-ice puck. Newton's first law brings the eerie motion of the puck from the realm of the unnatural to that of the natural. The first law can be stated as follows:

> Every object continues in its state of rest or of uniform rectilinear motion unless acted upon by an unbalanced force.

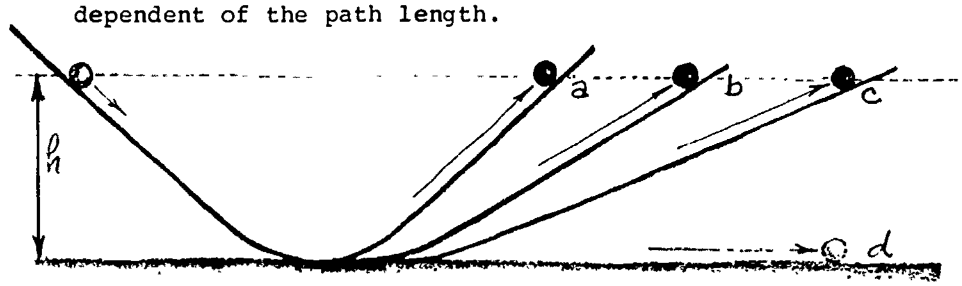
One must think of all the forces acting on an object. If all forces, including friction, balance, the body will be moving at constant $\vec{v}$ ("rest" being a special case, namely $\vec{v} = 0$). Straight-line motion is assured if all forces on the object balance or cancel.

Although Newton was the first to express this law in general terms, it was clearly anticipated by Galileo. Of course, neither Galileo nor Newton had dry ice pucks, so they could not experimentally observe motion for which friction had been so significantly reduced. Instead, Galileo did a thought experiment in which he imagined the friction to be zero.

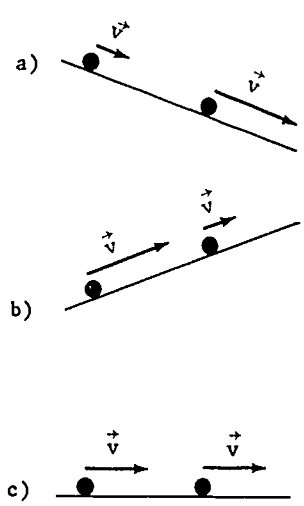
This thought experiment started with an actual observation. If a pendulum bob on the end of a string is pulled back and released from rest, it will swing through an arc rising to very nearly its starting height. Indeed, as Galileo showed, the pendulum bob will rise almost to its starting height even if a peg is used to change the path



From this observation he generates his thought experiment. A ball released from a height, h, on a frictionless incline, will roll up an adjoining incline, also frictionless, to the same height. Further, he reasons, this result is independent of the path length.



Another thought experiment. (a) A ball rolls down a smooth inclined plane; it gathers speed, i.e., $\vec{v}$ increases. (b) If it is made to roll up an incline, $\vec{v}$ decreases. (c) If the surface slants neither up nor down, i.e., is perfectly level, the ball, once started, will neither speed up nor slow down, i.e., $\vec{v}$ will remain constant.



The implications of the thought experiment illustrated in the above diagram are these: as the incline on the right is lowered from positions (a) to (b) and to (c), the ball must roll further in each case to reach its original height. In the final position (d) the ball can never reach its original height; therefore, Galileo believed the ball would roll in a straight line and at a uniform speed forever.

This tendency of objects to maintain their state of rest, or of uniform motion, is called "the principle of inertia." In fact, Newton's first law is sometimes referred to as the law of inertia. Inertia is an inherent property of all objects: the greater the inertia of an object, the greater is its resistance to a change in its state of motion. Material bodies are, so to speak, subject to a stubborn streak of nature so far as their state of motion is concerned: they continue to move with unchanging velocity (unchanging speed and direction) unless compelled by some force to do otherwise.

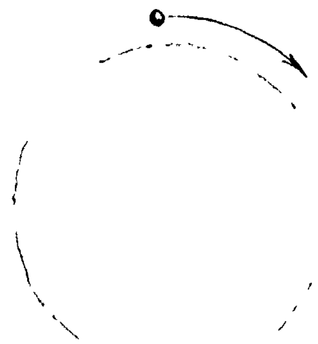
A consequence of Newton's first law is that if an object moves with a constant speed in a straight line, the forces acting on it are balanced. But wait. In the last section (3.5) we established that an object remains at rest as long as there are no unbalanced forces acting on it. Does this mean that the state of rest and the state of uniform motion are equivalent for dynamics? Indeed it does.

Do you understand inertia? See Study Guide 3.4.

This equivalence can be demonstrated by re-examining the quarrel over the toy. Suppose the quarrelling children were sitting on the deck of a barge that was slowly drifting, with uniform velocity, down a lazy river. Two observers—one on the barge and one on the shore—will give reports on the incident as viewed from their frames of reference. The observer on the barge will observe that the forces on the toy are balanced and will report that with respect to him it is at rest. The observer on the shore will report that the forces on the toy are balanced and that with respect to him the toy is in uniform motion. Which observer is right? They are both right. Rest or uniform motion depends on one's point of view.

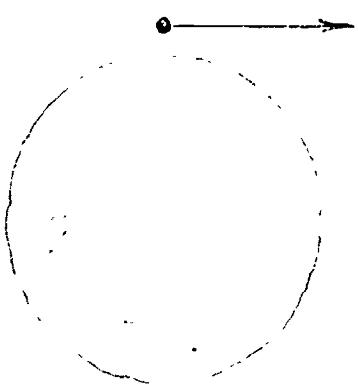
You may have found Galileo's thought experiment to be convincing, but remember that neither Galileo nor Newton proved the principle of inertia. Think of how you might try to verify that principle experimentally. You could start an object moving (perhaps a dry ice disc) in a situation in which there is no unbalanced force acting on it. Then you could observe whether or not the object continued to move uniformly in a straight line, as the first law claims it should. But there are at least two drawbacks to this experiment:

1. How do you know that there is no unbalanced force acting on the object? The only answer we have is: because the object continues to move uniformly in a straight line. But that reason is merely a restatement of the first law which we wanted to prove by experiment. Surely we cannot use the first law to verify the first law!



Galileo's idea of a straight line.

2. What is meant by a "straight line"? When Galileo wrote about uniform motion along a "straight" line, he was thinking of a line parallel to the earth's surface. Thus, an object given an initial push would move around and around the earth. Galileo's "straight line" was really a circle! Newton, on the other hand, meant an ideal straight line such as you talk about in plane geometry. Such a line is not parallel to the earth's surface; instead it continues indefinitely out into space. The line traced by a dry ice disc in any actual laboratory experiment is so short relative to the earth's circumference, that it could fit either Galileo's or Newton's meaning.



Newton's idea of a straight line.

What, then, is the significance of Newton's first law of motion? For convenience let us list the important insights the first law provides.

1. It presents inertia, the tendency of an object to maintain its state of rest or uniform motion, as a basic inherent property of all objects.

2. It makes no dynamical distinction between an object at rest and an object in uniform motion. Both states are characterized by the absence of unbalanced forces.

3. It raises the whole issue of reference frames. An object stationary for one observer might be in motion for another observer; therefore, if the ideas of rest or uniform motion are to have any significance, a reference frame must be stipulated.

4. It is a general law. It emphasizes right from the start that a single scheme is being formulated to deal with motion anywhere in the universe. For the first time no distinction is made between terrestrial and celestial domains. The same law applies to objects on earth as for planets and stars.

Study Guide 3.5 and 3.6

5. The first law informs us of the behavior of objects when no force acts on them. Thus, it sets the stage for the question: what happens when a force does act on an object?

Q6 Can you give a Newtonian explanation of a dry-ice puck's uniform motion across the table top?

Q7 How does Newton's concept of inertia differ from Galileo's?

3.7 Newton's second law of motion. So far we have met two of our three objectives: the explanation of rest and of uniform motion. In terms of the first law, the states of rest and uniform motion are equivalent; they are the states that result when no unbalanced force acts on an object.

The last section was concluded by a list of insights provided by the first law. Perhaps you noticed that there was no quantitative relationship established between force and inertia. Newton's second law of motion enables us to reach our third objective—the explanation of acceleration—and also provides a quantitative relationship between force and inertia. We shall study these two aspects of the second law, force and inertia, individually. First we consider the situation in which different forces act on the same object, and then the situation in which the same force acts on different objects.

To emphasize the force aspect, the second law can be stated as follows:

> The net unbalanced force acting on an object is directly proportional to, and in the same direction as, the acceleration of the object.

More briefly, this can be written as:

acceleration is proportional to net force.

If we let $\vec{F}$ stand for force and, as before, let $\vec{a}$ stand for acceleration, we can rewrite this as:

$\vec{a}$ is proportional to $\vec{F}$.

To say that one quantity is proportional to another is to make a precise mathematical statement. Here it means that if a given force causes an object to move with a certain acceleration, then twice the force will cause the same object to have twice the acceleration, three times the force will cause three times the acceleration, and so on. Using symbols, this becomes:

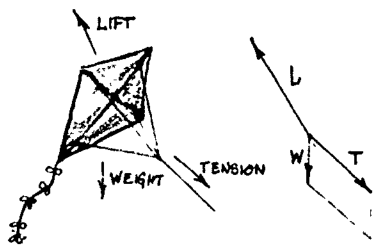
if $\vec{F}$ causes	$\vec{a}$
then $2\vec{F}$ will cause	$2\vec{a}$
$3\vec{F}$ will cause	$3\vec{a}$
$\frac{1}{2}\vec{F}$ will cause	$\frac{1}{2}\vec{a}$
$5.5\vec{F}$ will cause	$5.5\vec{a}$

and so on. So much for the effect of different forces on a single object. Now we can consider the inertia aspect of the second law, the effect of the same force acting on different objects. In discussing the first law, we defined inertia as the resistance of an object to having its velocity changed. We know from experience and observation that some objects have greater inertia than others. For instance, if you were to throw a baseball and a shot put with your full force, you know very well that the baseball would be accelerated to a greater speed than the shot put. The acceleration given to a body thus depends as much on an inherent characteristic as it does on the force applied.

Find the net force on the body in each case:

Apple falling - negligible friction

Feather falling at nearly constant speed



Kite, held suspended by wind

Man running against wind

(Remember: the force referred to is always the unbalanced or resultant force.)

Inertia seems to be associated with the massiveness, the amount of matter in an object. These vague ideas of "massiveness" and "amount of matter" have been replaced in physics by the quantitative concept of mass. Mass is a measure of the inertia of any object.

If you have several objects, and if you apply the same force to each, the various accelerations will not be the same. Newton's claim is that the resulting acceleration of each object is inversely proportional to its mass. Using m as a symbol for mass (a scalar quantity), and a as a symbol for the magnitude of the vector acceleration $\vec{a}$, we can write

What does it mean to say that mass is a scalar quantity?

a is inversely proportional to m,

or, what is mathematically the same thing,

$$a \text{ is proportional to } \frac{1}{m} .$$

This means that if a certain force causes a given object to have a certain acceleration, then the same force will cause: an object having twice the mass to undergo one-half the acceleration; an object having three times the mass, one-third the acceleration; an object of one-fifth the mass, five times the acceleration; and so on. Using symbols, we can express this as:

if for a given force $\vec{F}$

m will experience	a,
then 2m will experience	1/2 a,
3m will experience	1/3 a,
1/5 m will experience	5a,
2.5m will experience	0.4a,
etc.	

Complete this table of the relationships between mass and acceleration for a fixed force:

mass	acceleration
m	30 m/sec^2
2m	15 m/sec^2
3m	
1/5m	
0.4m	
45m	
	3 m/sec^2
	75 m/sec^2

Now we can combine the roles played by force and mass in the second law into a single statement:

> The acceleration of an object is directly proportional to, and in the same direction as, the unbalanced force acting on it, and inversely proportional to the mass of the object.

Fortunately, the idea expressed in this long statement can be summarized by the equation

$$\vec{a} = \frac{\vec{F}_{net}}{m}$$

Study Guide 3.7

Study Guide 3.8

which can be taken as a statement of Newton's second law. In this expression the unbalanced or net force is symbolized by $\vec{F}_{net}$. The second law may of course equally well be written in the form

$$\vec{F}_{net} = m\vec{a}.$$

This is probably the most fundamental single equation in all of Newtonian mechanics. We must not let the simplicity of the law fool us; behind that equation there is a lengthy "text."

If the law is to be useful, however, we must find a way to express force and mass numerically. But how? By measuring the acceleration which an unknown force gives a body of known mass, we could compute a numerical value for the force. Or, by measuring the acceleration which a known force gives a body of unknown mass, we could compute a numerical value for the mass. But we seem to be going in a circle in trying to find values for force and mass—to find one we apparently need to know the other in advance.

One straightforward solution to this dilemma is to choose some convenient stable object, perhaps a certain piece of polished rock or metal, as the universal standard of mass. We arbitrarily assign it a mass of one unit. Such a standard object has, in fact, been agreed on by the scientific community. Once this unit has been selected we can proceed to develop a measure of force.

Although we are free to choose any object as a standard of mass, ideally it should be exceedingly stable, easily reproducible, and of reasonably convenient magnitude.

By international agreement, the primary standard of mass is a cylinder of platinum-iridium alloy, kept near Paris at the International Bureau of Weights and Measures. The mass of this platinum cylinder is defined as 1 kilogram.

Accurate copies of this international primary standard of mass have been deposited at the various standards laboratories throughout the world. From these, in turn, other copies are made for distribution to manufacturers and laboratories.

Now we can decide on an answer to the question of how much "push or pull" should be regarded as one unit of force. We will simply define 1 unit force as a force which when acting alone causes a mass of 1 kilogram to accelerate at the rate of 1 meter/second^2.

Imagine an experiment in which the standard 1-kilogram mass is pulled in a horizontal direction across a level, frictionless surface with a light cord, and the pull is regulated so that the 1-kilogram mass accelerates at exactly 1 m/sec^2. The force will be one unit in magnitude.

What shall we call this unit of force? According to the second law (using only magnitudes):

$$F = ma$$

1 unit of acceleration = 1 m/sec².

$$1 \text{ unit of force} = 1 \text{ unit of mass} \times 1 \text{ unit of acceleration}$$

$$= 1 \text{ kilogram} \times \frac{1 \text{ meter}}{\text{second}^2}$$

$$= 1 \frac{\text{kilogram} \times \text{meter}}{\text{second}^2}$$

$$= 1 \frac{\text{kg} \times \text{m}}{\text{sec}^2}.$$

1 unit of force = 1 kg × m/sec².

Thus, $1 \frac{\text{kg} \times \text{m}}{\text{sec}^2}$ of force is that quantity of force which causes a mass of 1 kg to accelerate 1 m/sec².

The unit kg × m/sec² has been given a shorter name: it is called the newton (abbreviated as N). The newton is, therefore, a derived unit which is defined in terms of a particular relationship between the meter, the kilogram and the second. These three are taken as the fundamental units of the mks system of units.

Study Guide 3.10 and 3.11

Q8 A net force of 10 N gives an object a constant acceleration of 4 m/sec². The object's mass is _________?

Q9 Newton's second law holds only when frictional forces are absent. (True or false.)

Q10 A 2 kg object is shoved across the floor with an initial speed of 10 m/sec. It comes to rest in 5 sec.

(a) What was the average acceleration?

(b) What was the magnitude of the force producing this acceleration?

(c) What do we call this force?

3.8 Mass, weight and gravitation. Objects may be acted upon by all kinds of forces—by a push of the hand, by a pull on a string, or by a blow from a hammer. These forces don't have to be "mechanical" or exerted by contact only, they can be due to gravitational, electric, magnetic or other actions. The laws of Newton are valid for all of them.

The force of gravity, which we take so much for granted, is of the kind that acts without direct contact, not only on a stone or ball that is falling near the earth, but also across empty space, for example on one of the artificial satellites around it.

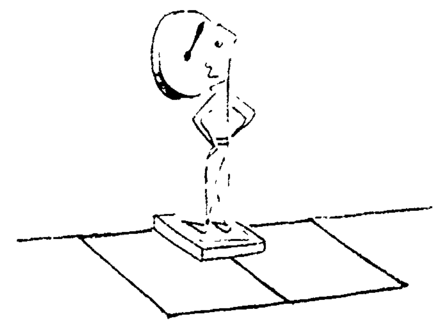
We shall give the gravitational force which pulls all objects towards the earth the symbol $\vec{F}_g$. The magnitude of the gravitational pull, $\vec{F}_g$, on any particular object is, roughly speaking, the same anywhere on the surface of the earth. When we choose to be more precise, we can take into account the following facts:

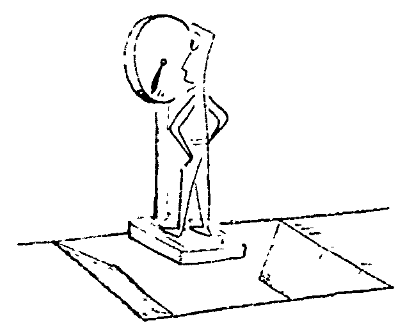
1. the earth is not exactly spherical and

2. there are irregularities in the composition of the earth's crust. These two factors cause slight variations in the gravitational force as we go from place to place. (Ge-

ologists make use of these variations in locating oil and mineral deposits.)

The term weight is used frequently in every day conversation as if it is the same as bulk or mass. In physics, we define the weight of an object as equal to the gravitational force that the body experiences. Hence weight (symbol $\vec{W}$) is a vector, as are all forces, and $\vec{W} = \vec{F}_{grav}$, by definition. When you stand on a bathroom scale to "weigh" yourself, your weight is the downward force the planet exerts on you. The bathroom scale is only registering the force with which it is pushing up on your feet and to keep you in balance, and this will be equal in magnitude to your weight if the scale is fixed and is not accelerating. If, on the other hand, the gravitational force on you is the <u>only</u> force you experience, and there is no other force on you that balances it, then you must be in free fall motion!





This is what would happen if the bathroom floor suddenly gave way as you stand on the scale (forgive the absurd "thought experiment"!). You and the scale would both fall down at equal rates, as all bodies do, pulled down by their weight. Your feet would now barely touch the scale, if at all; and if you looked down you would see that the scale registers zero since it is no longer pushing up on you. This does not mean you have lost your weight—that could only happen if the earth suddenly disappeared, or if you are taken to interstellar space. No, $\vec{F}_{grav}$ acts as before, and keeps you in free fall; it just means a bathroom scale does not measure your weight if it is accelerating.

We are now in a position to deepen our insight into Galileo's experiment on falling objects. Galileo's experiments indicated that every object (at a given locality) falls with uniform acceleration. And what causes a uniform acceleration? A constant net force—in this case, in free fall, just $\vec{F}_g$ or $\vec{W}$. Newton's second law gives us the relationship between this force and the resulting acceleration and we can write

Study Guide 3.15

$$\vec{F}_g = m\vec{a}_g$$

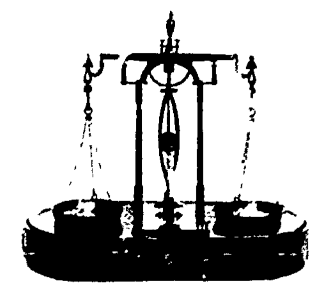
where m is the mass of the falling object and $\vec{a}_g$ is the acceleration resulting from the gravitational force $\vec{F}_g$. Thus we would conclude from Newton's second law that as long as the gravitational force is constant, the resulting acceleration is constant.

Galileo, however, did more than just show that all objects fall with uniform acceleration: he showed that all objects fall with the same uniform acceleration. Regardless of an object's weight, $\vec{F}_g$, it falls with the same acceleration $\vec{a}_g$. Is this consistent with the above relation, $\vec{F}_g = m\vec{a}_g$? It is only consistent provided there is a direct proportionality between weight and mass: if m is doubled, $\vec{F}_g$ must double, if m triples, $\vec{F}_g$ must triple. This is a profound result indeed: weight and mass are entirely different concepts—

weight is the gravitational force on an object (hence weight is a vector)

mass is a measure of the resistance of an object to changes in motion, a measure of inertia (mass is a scalar)

—yet the magnitudes of these two quite different quantities are proportional in a given locality.



What does it measure--mass or weight?

As a specific example, let us compare the magnitude of the weight and the mass of a 1-kg object and a 2-kg object. The respective weights $\vec{W}_1$ and $\vec{W}_2$, can be computed as follows (at the surface of the earth):

$$\vec{W}_1 = \vec{F}_g = m_1\vec{a}_g = (1\text{ kg})(9.8\text{ m/sec}^2) = 9.8\text{ N}$$

$$\vec{W}_2 = \vec{F}_g = m_2\vec{a}_g = (2\text{ kg})(9.8\text{ m/sec}^2) = 19.6\text{ N}$$

We see again that the ratio of the magnitudes of the weights (19.6:9.8 = 2:1) is the same as the mass ratio. These ratios will only be the same, however, if both objects are at the same location. For example, if the 1-kg object is placed at a higher altitude, its weight will be diminished but its mass will not; and so the weight ratio between it and the 2-kg object will change while the mass ratio remains 2:1. In other words, weight depends on position, but mass does not.

Study Guide 3.17

Q11 An astronaut is orbiting the earth in a space capsule. The acceleration of gravity is half its value on the surface of the earth. Which of the following statements is true?
(a) His weight is zero.
(b) His mass is zero.
(c) His weight is half its original value.
(d) His mass is half its original value.
(e) His weight is the same.
(f) His mass is the same.

Q12 A boy jumps from a table top. Halfway between the table top and the floor, which of these statements (for Q11) is true?

3.9 Newton's third law of motion. Newton's first law describes motion of objects when they are in a state of equilibrium, that is, when the resultant force acting on them is zero. The second law tells how their motion changes when the resultant force is not zero. Neither of these laws indicates what the origin of the force is.

For example, in the 100-meter dash, an Olympic track star will go from rest to nearly his top speed in a very short time. With high speed photography his initial acceleration could be measured. Also, we could measure his mass. With mass and acceleration known, we could use $\vec{F} = m\vec{a}$ to find the force acting on him. But where does the force come from? It must have something to do with the runner himself, but can he exert a force on himself as a whole? Can you lift yourself by your own bootstraps?

Newton's third law of motion helps us to explain just such puzzling situations. First, let us examine the third law to see what it claims. In Newton's words,

> To every action there is always opposed an equal reaction: or, mutual actions of two bodies upon each other are always equal and directed to contrary parts.

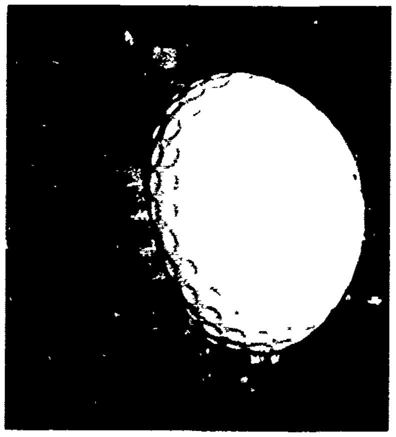
This is a rather literal translation. It is generally agreed, however, that the word force may be substituted for both the word action and the word reaction in Newton's statement.

The Principia was written in Latin, although in Newton's day scholars were beginning to use their native language more and more in their writings. The English language itself has always been changing, and so what constitutes the most accurate translation of seventeenth-century Latin into twentieth-century English is not beyond dispute.

The most startling idea to come out of this statement is that forces always exist in pairs. Indeed, any thought of a single unaccompanied force is without any meaning whatsoever. On this point Newton wrote:

> Whatever draws or presses another is as much drawn or pressed by that other. If you press a stone with your finger, the finger is also pressed by the stone.

This suggests that forces always arise as a result of interactions between objects: object A pushes or pulls on B while at the same time object B pushes or pulls with precisely equal amount on A. These paired pulls and pushes are always equal in magnitude and opposite in direction.



In the collision between the ball and the club, the force the ball exerts on the club is equal and opposite to the force the club exerts on the ball.

The terms action and reaction are arbitrary, as is the order of their naming. The action does not cause the reaction. The two coexist. And most important, they are not acting on the same body. In a way, they are like debt and credit: one is impossible without the other; they are equally large but of opposite sign, and they happen to two different objects.

We can describe the situation where A exerts a force on B and at the same time B exerts on A an equal and opposite force. In the efficient shorthand of algebra we may write

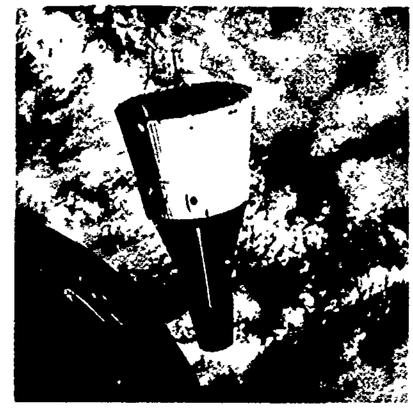
$$\vec{F}_{AB} = -\vec{F}_{BA}$$

This is the equivalent of Newton's statement that,

> Whenever two bodies interact, the forces they exert on each other are equal in magnitude and opposite in direction.

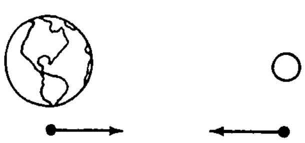
Note, now, what the third law does not say—for this, too, is of importance. It does not speak of how the push or pull is applied, whether it is through contact (if we could

define that word) or by magnetic action or electrical action. Nor does the law require that the force be either an attraction or repulsion. The third law really does not depend on any particular kind of force. Indeed, what makes the third law extremely valuable, is its universal nature.



Study Guide 3.19

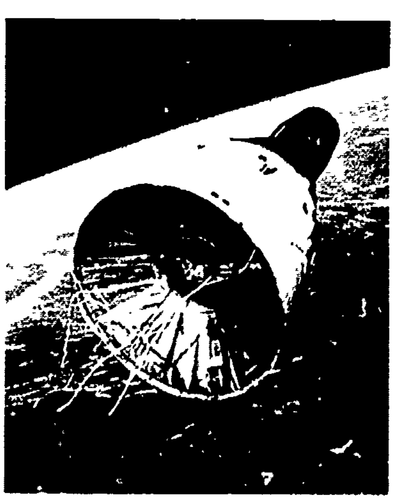
Q13 A piece of fishing line will break if the force exerted on it is greater than 500 N. Will the line break if two people at opposite ends of the line pull on it, each with a force of 300 N?



The force on the moon due to the earth is equal and opposite to the force on the earth due to the moon.

3.10 Using Newton's laws of motion. We have discussed Newton's three laws of motion in some detail. In doing so, we saw that each law is important in its own right. The first law emphasizes the modern point of view for the study of motion: what requires explanation is not uniform motion, but change of motion. The first law stresses that what one must account for is why an object speeds up or slows down or changes direction. The second law asserts that the rate of change of velocity of an object is related to both the mass of an object and the net force applied to it. In fact, the very meanings of force and mass are bound up in the second law. The third law is a statement of the force relationship between interacting objects.

Despite their individual importance, Newton's three laws are most powerful when used together to explain complex phenomena. Let us examine a few specific examples that illustrate the use of these laws.



Example 1. On Monday, September 12, 1966, a dramatic experiment based on Newton's second law was carried out high over the earth. In this experiment, the mass of an orbiting Agena rocket was determined by accelerating it with a push from a Gemini spacecraft. After the Gemini spacecraft made contact with the Agena rocket, the aft thrusters on the Gemini, calibrated to give a thrusting force of 890 N, were fired for 7 sec. The change in velocity of the spacecraft and rocket was 0.93 m/sec. The mass of the Gemini spacecraft was 3360 kg. What was the mass of the Agena?

A force of magnitude 890 N acts on a total mass M where

$$M = M_{Gemini} + M_{Agena} \text{ or } M = 3360 \text{ kg} + M_{Agena} .$$

The magnitude of the acceleration is given as follows:

$$a = \frac{\Delta v}{\Delta t}$$

$$= \frac{.93 \text{ m/sec}}{7 \text{ sec}}$$

$$= 0.13 \text{ m/sec}^2 .$$

Newton's second law gives us the relation

$$T = Ma$$

In this equation, T is used for force since it is a thrusting force.

or

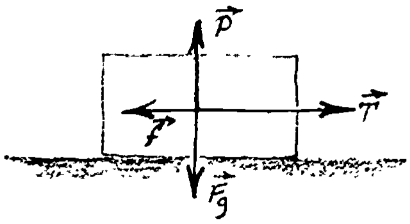
$$T = (M_{Agena} + 3360 \text{ kg})a$$

where T is the thrust. Solving for M_{Agena} gives

$$M_{Agena} = \frac{T}{a} - 3360 \text{ kg} = \frac{890 \text{ N}}{.13 \text{ m/sec}} - 3360 \text{ kg}$$

$$M_{Agena} = 6850 \text{ kg} - 3360 \text{ kg} = 3490 \text{ kg}.$$

The mass of the Agena was known to be approximately 3660 kg which means there was a 5% error in the measurement. (This experiment was performed to determine the feasibility of this technique as a means for finding the mass of a foreign satellite while it is in orbit.)



Example 2. A case of books whose mass is 8.0 kg rests on a table. What constant horizontal force $\vec{T}$ is required to give it a velocity of 6 m/sec in 2 sec, starting from rest, if the friction force $\vec{f}$ between the moving case and the table is constant and is equal to 6 newtons? (Assume all forces act at the center of the case.)

In solving problems such as this, it is always helpful to make a sketch showing the forces acting. The forces acting on the case are the frictional force $\vec{f}$, the force $\vec{T}$, the force of gravity $\vec{F}_g$ (the case's weight), and the force of the table on the case $\vec{P}$. The case pushes down on the table with a force $\vec{F}_g$ and the table pushes back on the case with a force $\vec{P}$ (Newton's third law). Therefore

$$\vec{F}_g = -\vec{P}$$

What is your interpretation of this equation?

or

$$\vec{F}_g + \vec{P} = 0.$$

For the forces parallel to the surface we can write

$$\text{unbalanced force} = \vec{T} - \vec{f} = \vec{T} - 6\text{N}$$

From the second law we have

$$\vec{F}_{net} = m\vec{a}$$

$$\vec{T} - 6\text{N} = m\vec{a}$$

$$\vec{T} = m\vec{a} + 6\text{N}$$

The magnitude of the acceleration is

$$a = \frac{\Delta v}{\Delta t} = \frac{6\ \text{m/sec}}{2\ \text{sec}} = 3\ \text{m/sec}^2.$$

Thus we can now determine the magnitude of the horizontal force.

Study Guide 3.13, 3.14, 3.16, 3.18

$$T = ma + 6.0\ \text{N} = (8.0\ \text{kg})\ (3\ \text{m/sec}^2) + 6.0\ \text{N}$$

$$T = 24\ \text{N} + 6\ \text{N} = 30\ \text{N}.$$

3.11 Nature's basic forces. Our study of Newton's laws of motion has increased our understanding of objects at rest, in uniform motion, and accelerating. However, we have accomplished much more in the process. Newton's first law alerted us to the importance of reference frames. What you observe depends upon your point of view—your frame of reference. A critical analysis of the relationship between descriptions from different frames of reference was a forerunner of the theory of relativity.

Newton's second law alerted us to the importance of forces. In fact, it presents us with a mandate: when we observe acceleration, find the force! For example, when we recognize that an orbiting satellite is accelerating, we look for a force. We might begin this search by giving the force a name, for example, gravitational force.

But a name alone adds nothing to our basic understanding. We really want to know what determines the force acting on a satellite. Does the force depend on the earth? Obviously it does. Does the force depend on the satellite's position? On its velocity? On the time? Answers to questions such as these can be summarized in a force law which describes the force in terms of those factors it depends on. A force law provides a basis for understanding the way in which the earth and a satellite interact with each other. Knowing the force law, the physicist claims to "understand" the nature of the interaction.

Gravitational attraction is just one of the basic ways in which objects interact. It is exciting to realize that there appear to be very few of these basic interactions. In fact, physicists now believe there are just four. Does it surprise you to think there are so few? Imagine—all we observe in nature is the consequence of just four basic interactions. In terms of our present understanding, all the forces of nature—subnuclear and nuclear, atomic and molecular, terrestrial and solar, galactic and extragalactic—are the manifestations of these four basic interactions.

There is, of course, nothing sacred about the number four. The number might be reduced or enlarged due to new discoveries. In fact, physicists hope that as they gain further insight into these basic interactions, two (or more) of them might be seen as the consequence of something even more basic.

The first interaction, the gravitational, becomes important only on a very large scale, when there are tremendous amounts of matter involved. It literally holds the universe together. The second interaction concerns electric and magnetic processes. These processes are most important on a small scale—the atomic and molecular scale. We know the force laws governing gravitational and electromagnetic interactions; therefore they are fairly well "understood." The situation changes completely when we consider the remaining basic interactions. They are still the subject of vigorous research today. The third interaction (the so-called "strong" interaction) somehow holds the nucleus together.

"The Starry Night", 1889, by Vincent van Gogh.
Collection, The Museum of Modern Art, New York.

The fourth interaction (the so-called "weak" interaction) governs certain reactions among subnuclear particles.

We do, of course, have other names for forces. One of the most common, yet one of the least understood, is the frictional force. This subtle force fooled people for centuries into thinking that an object required a "pusher" or "puller" if it was to remain in motion. Yet, the frictional force is undoubtedly an electrical type of force; that is, the atoms on the surfaces of the objects sliding or rubbing against each other interact electrically. In this case, too, we seem to be able to understand all our observations of nature in terms of just a few basic interactions.

We shall be encountering some of these ideas again. We shall meet the gravitational force in Unit 2, the electrical and magnetic forces in Unit 4 and the nuclear force in Unit 6. In all these cases remember that a force plays the same role regardless of its origin; that is, an object sensitive to the force will be accelerated.

The knowledge that there are so few basic interactions is both surprising and encouraging. It is surprising because at first glance the world seems so complicated; it is encouraging because our elusive goal—an understanding of nature—seems nearer.

The intuition that all of nature's phenomena are interlinked on a grand scale is shared by scientists as well as artists.

3.1 Newton's First Law: Every object continues in its state of rest or of uniform rectilinear motion unless acted upon by an unbalanced force.

Newton's Second Law: The acceleration of an object is directly proportional to, and in the same direction as, the unbalanced force acting on it, and inversely proportional to the mass of the object.

Newton's Third Law: To every action there is always opposed an equal reaction: or, mutual actions of two bodies upon each other are always equal and directed to contrary parts.

3.2 The Aristotelian explanation of motion should not be dismissed lightly. Great intellects of the Renaissance period such as Leonardo da Vinci, who, among other things, designed artillery for launching projectiles, apparently did not challenge the Aristotelian explanation. One reason for the longevity of Aristotle's ideas is that they are so closely aligned with our common-sense ideas. In what ways do your common-sense notions of motion agree with Aristotle's?

3.3 Three children, Karen, Keith and Sarah are each pulling on the same toy.

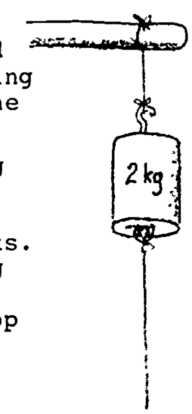
Karen pulls toward the east with a force of 8 units.

Sarah pulls toward the north with a force of 6 units.

Keith pulls in a direction 30° south of west with a force of 12 units.

a) Is there a net (i.e., unbalanced) force on the toy?
b) If there is a net force, what is its magnitude and direction?

3.4 A 2 kg mass is suspended by a string. A second string is tied to the bottom of the mass.



a) If the bottom string is pulled with a sudden jerk, the bottom string breaks.
b) If the bottom string is pulled with a steady pull, the top string breaks.

Explain.

(Try this experiment for yourself. You might tie thread to a brick, or a hammer, or a pipe wrench.)

3.5 In terms of Newton's first law, explain:

a) why people in a moving car lurch forward when the car suddenly decelerates;
b) what happens to the passengers of a car that makes a sharp and quick turn;
c) why, when a coin is put on a phonograph turntable and the motor is started, the coin flies off when the turntable reaches a certain speed. Why doesn't it fly off before?

3.6 a) You exert a force on a box, but it does not move. How would you explain this? How might an Aristotelian explain it?
b) Suppose now that you exert a greater force, and the box moves. Explain this from your point of view and from an Aristotelian point of view.

3.7 Assume that the floor of a laboratory could be made perfectly horizontal and perfectly smooth. A block of wood is placed on the floor and given a small push. Predict the way in which the block will move. How would this motion differ if the whole laboratory were moving with constant velocity during the experiment. How would it differ if the whole laboratory were accelerating along a straight line? If the block were seen to move in a curved path along the floor, how would you explain this?

3.8 A body is being accelerated by an unbalanced force. If the magnitude of the net force is doubled and the mass of the body is reduced to one-third of the original value, what will be the ratio of the second acceleration to the first?

3.9 Hooke's Law says that the force exerted by a stretched or compressed spring is directly proportional to the amount of the compression or extension. As Robert Hooke put it in announcing his discovery:

> ...the power of any spring is in the same proportion with the tension thereof: that is, if one power stretch or bend it one space, two will bend it two, three will bend it three, and so forward. Now as the theory is very short, so the way of trying it is very easie.

If Hooke says it's "easie," then it might well be so. You can probably think immediately of how to test this law using springs and weights. Try designing such an experiment; then after checking with your teacher, carry it out.

Hooke's experiment is described in his own words in W. F. Magie, A Source Book in Physics, McGraw-Hill, 1935.

3.10 If you have dynamics carts available, here is one way of doing an experiment to test the inverse proportionality between acceleration and mass:

a) Add load blocks to one or the other of two carts until the carts balance when placed on opposite platforms of a laboratory balance. Balance a third cart with one of the first pair. Each cart now has mass m. (State two main assumptions involved here.)

b) Accelerate one cart on a level surface using the rubber-band technique; that is, pull the car with the rubber band keeping it stretched a constant amount. Any other method can also be used that will assure you that, within reason, the same force is being applied each time. Record the position of the cart at equal time intervals by means of stroboscopic photography.

c) Repeat the last step in all details, but use two carts hooked together. Repeat again using all three carts hooked together. In all three cases it is crucial that the applied force be essentially the same.

d) Determine the value of acceleration for masses of m (1 cart), 2m (2 carts), and 3m (3 carts).

e) Prepare a graph of a vs. m, of a vs. $\frac{1}{m}$, and of $\frac{1}{a}$ vs. m.
Comment on your results.

3.11 Complete this table:

a)	1.0 N	1.0 kg	1.0 m/sec²
b)	24.0 N	2.0 kg	12.0 m/sec²
c)	N	3.0 kg	8.0 m/sec²
d)	N	74.0 kg	0.2 m/sec²
e)	N	0.0066 kg	130.0 m/sec²
f)	72.0 N	kg	8.0 m/sec²
g)	3.6 N	kg	12.0 m/sec²
h)	1.3 N	kg	6.4 m/sec²
i)	30.0 N	10.0 kg	m/sec²
j)	0.5 N	0.20 kg	m/sec²
k)	120.0 N	48.0 kg	m/sec²

3.12 Recount in detail what steps you must take (in idealized experimentation) to determine the unknown mass m (in kilograms) of a certain object if you are given nothing but a frictionless horizontal plane, a 1-kg standard, an uncalibrated spring balance, a meter stick, and a stopwatch.

3.13 A certain block is dragged with constant velocity along a rough horizontal table top, by means of a spring balance horizontally attached to it which reads 0.40 N, no matter what the velocity happens to be. This means that the retarding frictional force between block and table is 0.40 N and not dependent on the speed. When the block is given a constant acceleration of 0.85 m/sec^2, the balance is found to read 2.1 N. Compute the mass of the block.

3.14 A sled has a mass of 4440 kg and is propelled by a solid propellant rocket motor of 890,000 N thrust which burns for 3.9 seconds.

a) What is the sled's average acceleration and maximum speed?

b) The data source states that this sled has a maximum acceleration of 30g (=30×a_g). How can that be, considering the data given?

c) If the sled travels a distance of 1530 m while attaining a top speed of 860 m/sec (how did it attain that high a speed?!), what is its average acceleration?

3.15 Discuss the statement that while the mass of an object is the same everywhere, its weight may vary from place to place.

3.16 A 75 kg man stands in an elevator. What force does the floor exert on him when the elevator

a) starts moving upward with an acceleration of 1.5 m/sec^2?

b) When the elevator moves upward with a constant speed of 2.0 m/sec?

c) When the elevator starts accelerating downward at 1.5 m/sec^2?

d) If the man were standing on a bathroom (spring) scale during his ride, what readings would the scale have in parts a, b, and c?

e) It is sometimes said that the "apparent weight" changes when the elevator accelerates. What could this mean? Does the weight really change?

3.17 A replica of the standard kilogram is constructed in Paris and then sent to the National Bureau of Standards in Washington. Assuming that this secondary standard is not damaged in transit, what is

a) its mass in Washington,
b) its weight in Paris and in Washington. (In Paris, a_g=9.81 m/sec^2; in Washington, a_g=9.80 m/sec^2.)

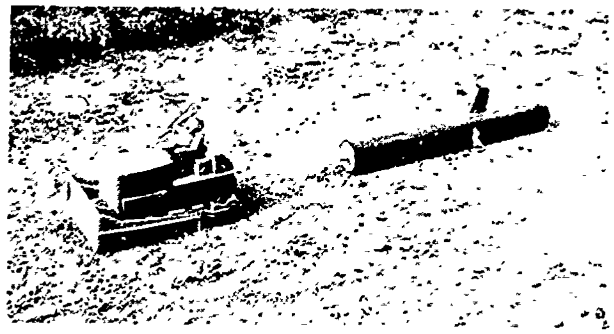
3.18 Consider the system consisting of a 1.0 kg ball and the earth. The ball is dropped from a short distance above the ground and falls freely. We can take the mass of the earth to be approximately 6.0×10^{24} kg.

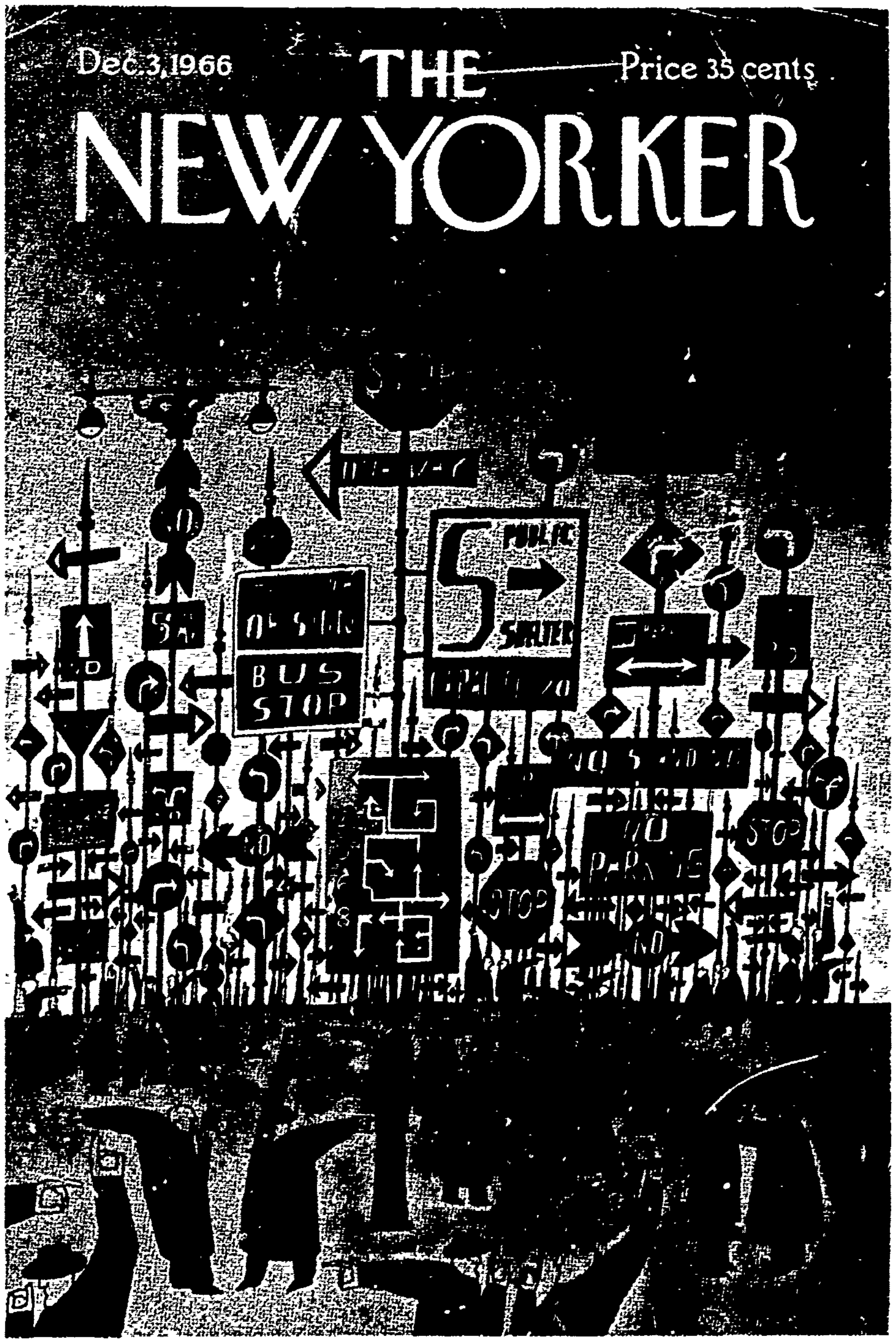
a) Make a vector diagram illustrating the important forces acting on the members of the system.
b) Calculate the acceleration of the earth in this interaction.
c) Find the ratio of the magnitude of the ball's acceleration to that of the earth's acceleration (a_b/a_e).

3.19 In terms of Newton's third law assess the following statements:

a) You are standing perfectly still on the ground; therefore you and the earth do not exert equal and opposite forces on each other.
b) The reason that a jet airplane cannot fly above the atmosphere is that there is no air to push against, as required by the third law.
c) The mass of object A is 100 times greater than that of object B, but even so the force it (A) exerts on B is no greater than the force of B on it.
d) C, D, and E are three objects having equal masses; if C and D both push against E at the same time, then E exerts only one-half as much force on C as C does on E.

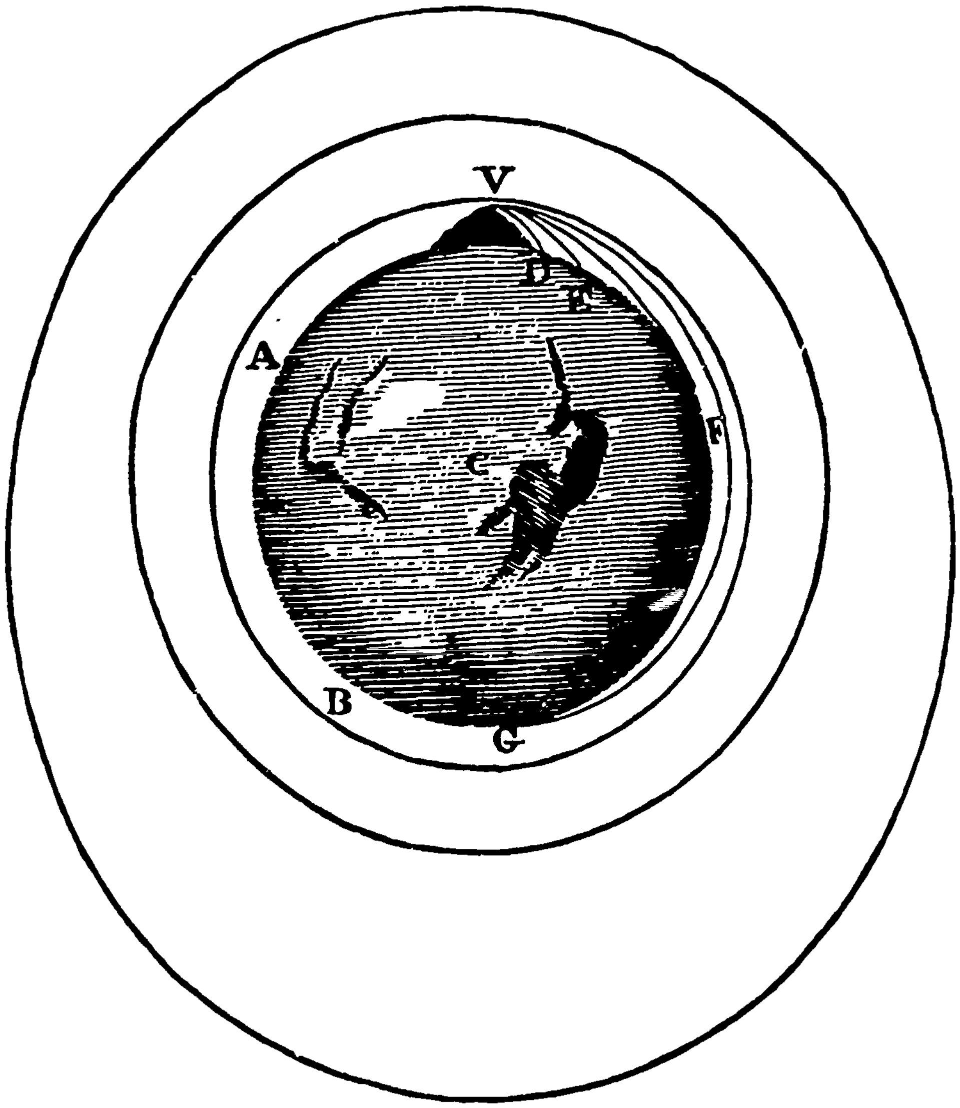
3.20 Consider a tractor pulling a heavy log in a straight line. On the basis of Newton's third law, one might argue that the log pulls back on the tractor just as strongly as the tractor pulls the log. But why, then, does the tractor move?





Drawing by Folon; © 1966 The New Yorker Magazine, Inc.

Chapter 4 Understanding Motion

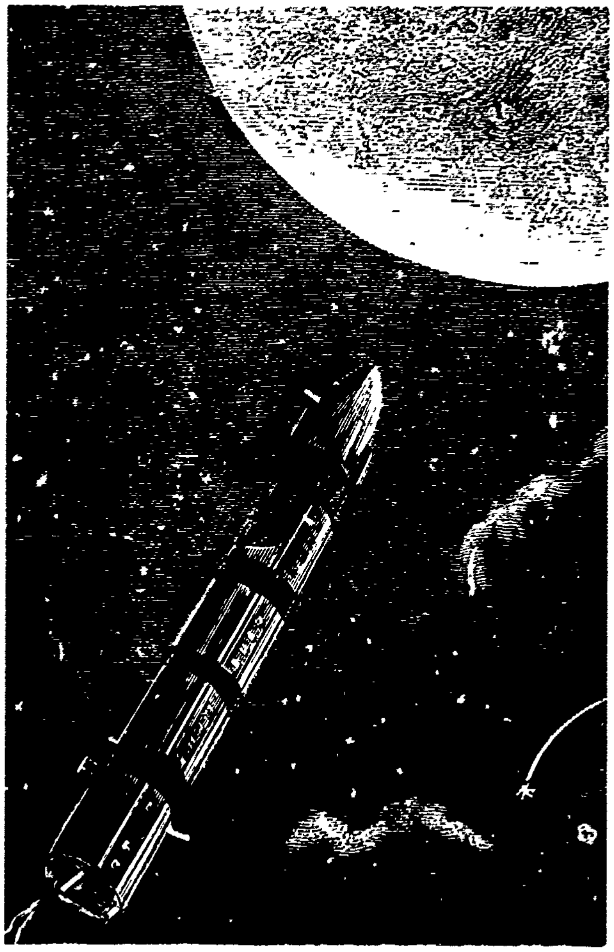


"...a stone that is projected is by the pressure of its own weight forced out of the rectilinear path, which by the initial projection alone it should have pursued, and made to describe a curved line in the air; and through that crooked way is at last brought down to the ground; and the greater the velocity is with which it is projected, the farther it goes before it falls to the earth. We may therefore suppose the velocity to be so increased, that it would describe an arc of 1, 2, 5, 10, 100, 1000 miles before it arrived at the earth, till at last, exceeding the limits of the earth, it should pass into space without touching it." [Newton's Principia]

4.1 A trip to the moon. Imagine a Saturn missile taking off in the early morning hours from its launching pad at Cape Kennedy. It climbs in a curved path above the earth, passing through the atmosphere and beyond. Successive stages of the missile burn out leaving finally an instrument capsule hurtling through the near vacuum of space toward its destination 240,000 miles away. Approximately 65 hours after take-off, the capsule circles the moon and plummets to its target—the center of the lunar crater Copernicus.

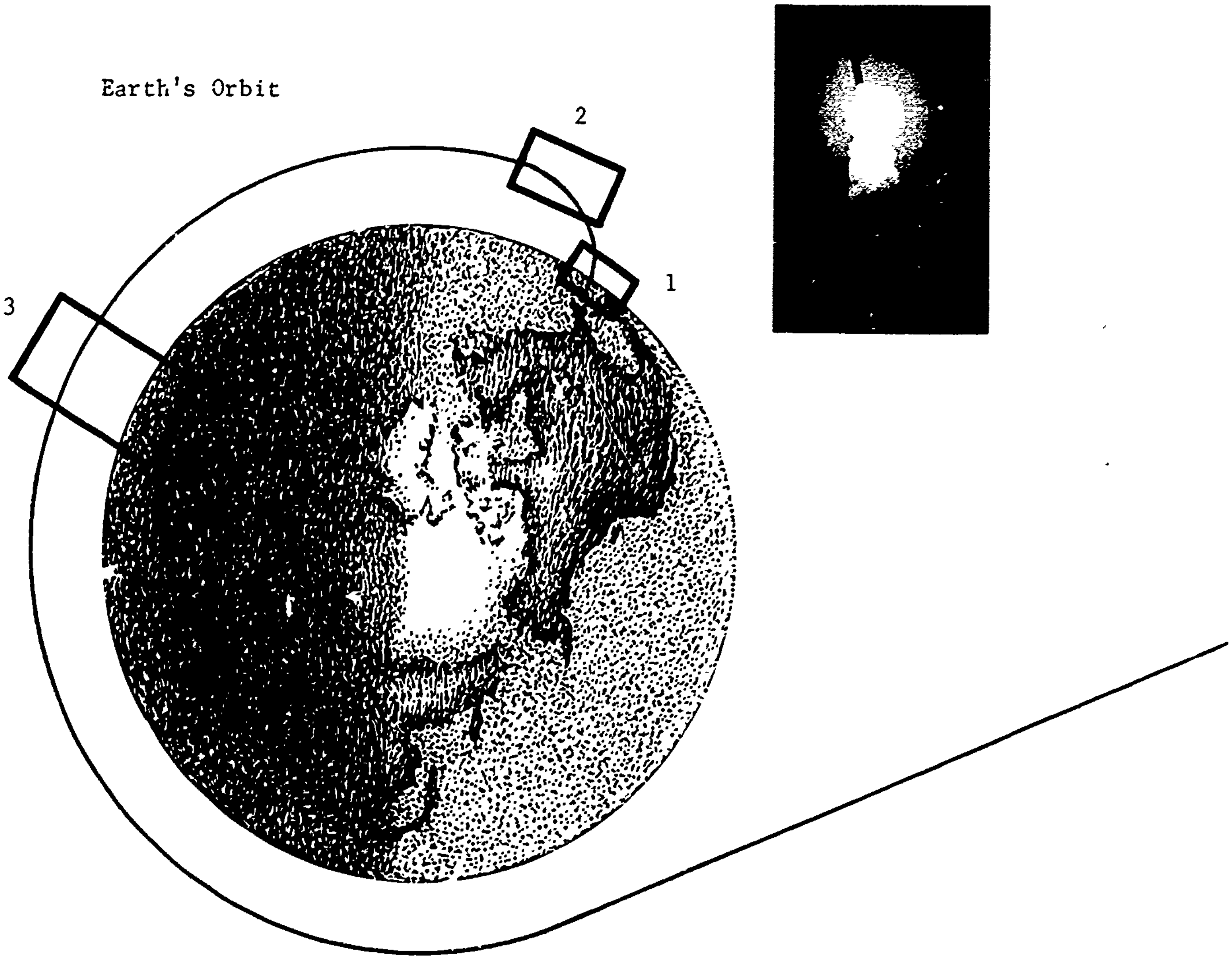
As you first think about it, you are likely to be struck by the complexity of such a voyage. The atmospheric drag at the beginning of the flight depends upon the missile's speed and altitude; the rocket's thrust changes with time. You must consider the changing gravitational pulls of the sun, the earth, and the moon as the capsule changes its position relative to them. Besides the forces, you must consider the facts that the rocket's mass is changing and that it is launched from a spinning earth, which in turn is circling the sun. Furthermore, the target—the moon—is moving around the earth at a speed of about 2,300 miles per hour.

The complexities of a rocket flight from earth to moon are indeed great and the amount of computation enormous—which is why NASA, the National Aeronautics and Space Administration, uses high speed electronic computers to help analyze and control the flight path. Though complicated in its totality, this flight can be broken down into small portions which are each relatively simple to analyze and describe. What we have learned in earlier chapters will be useful in this task.

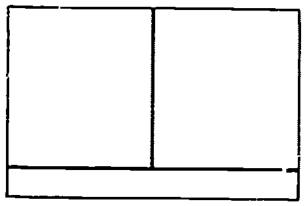


Over 100 years ago, the French author Jules Verne (1828-1905) portrayed how technology might be employed to place a man on the moon. In two prophetic novels, Verne launched three intrepid spacemen to the moon by means of a gigantic charge fixed in a steel pipe deep in the earth.

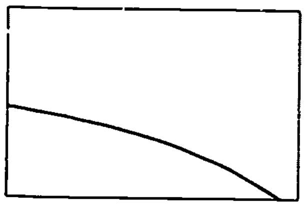
The world's first view of the earth taken by a spacecraft from the vicinity of the moon.



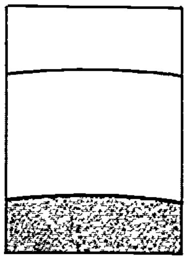
The earth-moon trip shown in the figure above can be divided into these six parts.



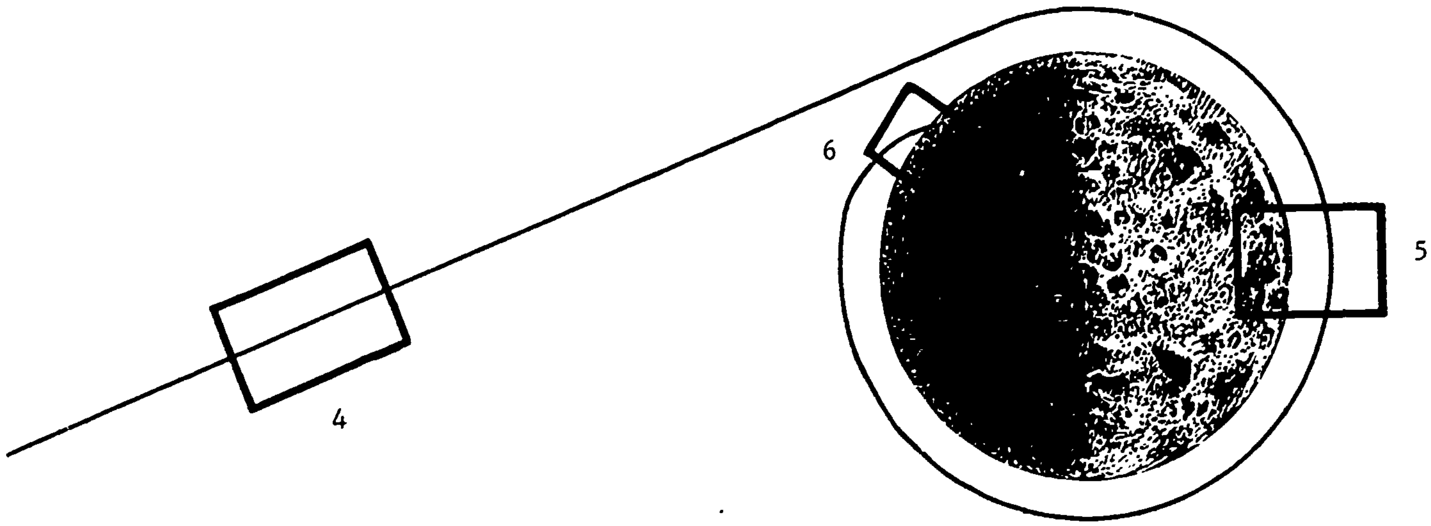
Part 1. The rocket accelerates vertically upward from the surface of the earth. The force acting on the rocket is essentially constant. The mass of the rocket, however, is decreasing. The value of the acceleration at any instant can be computed using Newton's second law.

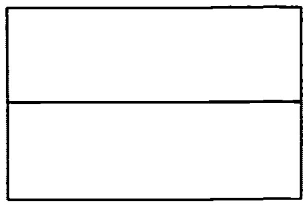


Part 2. The rocket, still accelerating, follows a curved path as it enters into an orbit about the earth.

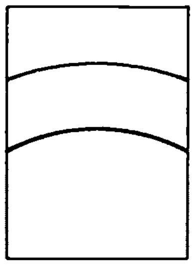


Part 3. In an orbit 115 miles above the earth's surface, the rocket circles at a constant speed of 17,380 miles/hr. The minimum escape velocity is 24,670 miles/hr; therefore, by accelerating in the direction of its path when it has reached the bottom of the semi-circular arc, the rocket can now thrust into distant space.

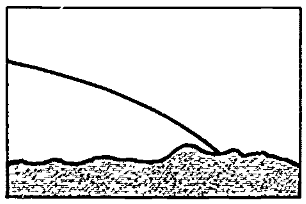




Part 4. In flight between earth and moon, only occasional bursts from the capsule's thrusters are required to keep it on course. Between these correction thrusts, the capsule moves under the influence of the gravitational forces of earth, moon, and sun. We know from Newton's first law that the capsule would move with constant velocity if it were not for these forces.



Part 5. The capsule is moving with constant speed of 1 mile/sec in a circular path 50 miles above the moon's surface.

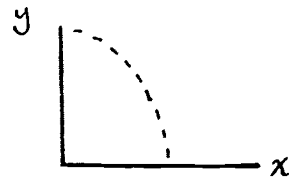


Part 6. The capsule is accelerating toward the surface of the moon. It follows an arching path before landing in the crater Copernicus.

Let us analyze in greater detail the last two parts of this trip—the capsule circling the moon and then falling to the moon's surface—since they are examples of circular motion and projectile motion, two important classes of motion. How shall we go about this? Must we travel to the moon, set up our cameras on the edge of the crater Copernicus, and make a stroboscopic record of the path of the capsule as it streaks through the lunar vacuum and crashes into the moon's surface? Not at all! We now realize, thanks to Galileo and Newton, that we can learn about the behavior of moving objects beyond our reach by studying the motion of objects near at hand.

4.2 Projectile motion. Imagine a rifle mounted on a tower with its barrel parallel to the ground. Imagine also that the ground over which the bullet will travel is level for a very great distance. Suppose further that at the instant a bullet leaves the rifle, a second, identical bullet is dropped from the same height as the muzzle of the rifle. The second bullet has no horizontal motion relative to the ground. Which bullet will reach the ground first? Do we need to know something about the muzzle velocity of the bullet and the height of the tower before we can answer this question?

To avoid confusion in notation, we let the displacement in the horizontal direction be x and the displacement in the vertical direction be y. This leads to the set of axes.



Consider first the motion of the bullet that is dropped. As a freely falling object, it accelerates toward the ground with uniform acceleration. Hence, in a time t it will fall a distance, y, given by

$$y = \frac{1}{2}a_g t^2$$

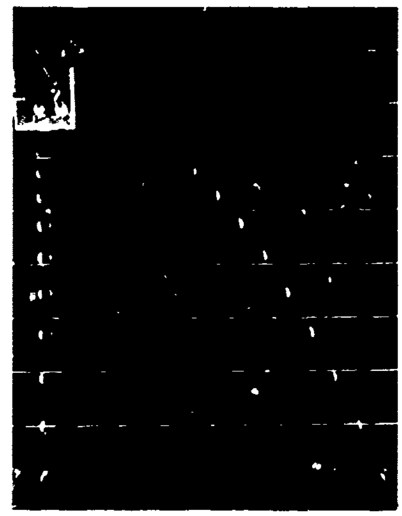
where a_g is the acceleration due to gravity.

The bullet that is fired horizontally from the rifle is an example of a projectile. Any object that is given an initial velocity and whose subsequent path is determined solely by the gravitational force and by the resistance of the air is a projectile. The path followed by a projectile is its trajectory. As the gunpowder explodes, the bullet is driven by the force of expanding gases and accelerated very rapidly until it reaches the muzzle of the rifle. On reaching the muzzle these gases escape and no longer push the bullet. At this moment, however, the bullet has a very large horizontal speed, v_x. The air will slow the bullet slightly, but we shall ignore that fact in our development

and imagine an ideal case with no air friction. As long as air friction is ignored, there is no net force acting in the horizontal direction. Hence, the horizontal speed will remain constant. From the instant the bullet leaves the muzzle, we would expect its horizontal motion to be described by the equation

$$x = v_x t.$$

As soon as the bullet leaves the gun, however, it becomes an unsupported body and falls toward the earth as it moves forward. Can we use the same equation to describe its fall that we used to describe the fall of the dropped bullet? That is, can we use $y = \frac{1}{2} a_g t^2$ to describe the fall of the high speed bullet? We believe, of course, that the bullet will fall to the ground, for any other answer would be contrary to our experience. But, whether it will fall at the same rate as the bullet with no horizontal motion is not clear. Nor, for that matter, can we be sure how falling will affect the bullet's horizontal motion. These doubts raise a more fundamental question that goes beyond just the behavior of bullets; namely, is the vertical motion of an object affected by its horizontal motion? Or vice versa?

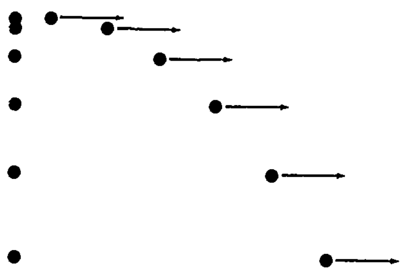


The two balls in this stroboscopic photograph were released simultaneously. The one on the left was simply dropped from rest position; the one on the right was given an initial velocity in the horizontal direction.

To answer these questions, we can carry out a real experiment similar to our thought experiment. We can use a special laboratory device designed to fire a ball in a horizontal direction at the moment that a second ball is released to fall freely from the same height. We set up our apparatus so that both balls are the same height above a level floor. The experiment is started. Although the motions of the balls may be too rapid for us to follow with the eye, we hear only a single sound as they strike the floor. This result suggests that the vertical motion of the projected ball is unaffected by its horizontal velocity.

Let us examine a stroboscopic photograph of this experiment. Equally spaced lines in the background aid our examination. Look first at the ball which was released without any horizontal motion. You see that it is accelerated because as it moves it travels a greater distance between successive flashes. Careful measurement of the photograph shows that the acceleration is uniform to within the precision of our measurements.

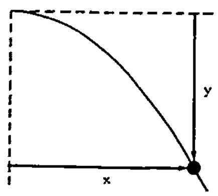
Now compare the vertical positions of the ball fired to the right with the vertical positions of the ball which is falling freely. The horizontal lines show that the distances of fall are the same for corresponding times. The two balls obey the same law for motion in a vertical direction. That is, at every instant they both have the same constant acceleration, a_g, the same downward velocity, and the same vertical displacement.

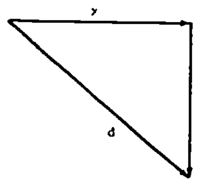


A projectile has a constant horizontal velocity v_x.

We can use the strobe photo to see if the downward acceleration of the projectile affects its horizontal velocity by measuring the horizontal distance between successive images. We see that the horizontal distances are essentially equal. Since the time intervals between images are equal, we can conclude that the horizontal velocity v_x is constant.

We now have definite answers to our questions. The horizontal motion of the ball does not interfere with the vertical motion, and vice versa. The two motions are completely independent of each other. This experiment can be repeated from different heights, and with different muzzle velocities, but the results will always show that the horizontal motion is independent of the vertical motion.





The displacement $\vec{d}$ of an object is a vector giving the straight-line distance from beginning to end of an actual path; $\vec{d}$ can be thought of as made up of a horizontal (x) and vertical (y) component of displacement, that is, $\vec{d} = \vec{x} + \vec{y}$ (added vectorially).

The independence of motions at right angles has interesting consequences. For example, it is easy to predict the displacement and the velocity of a projectile at any time during its flight. We need merely to consider the horizontal and vertical aspects of the motion separately and then add the results—vectorially. We can predict the positions x and y and the speeds v_x and v_y at any instant by application of the appropriate equations. For the horizontal component of motion

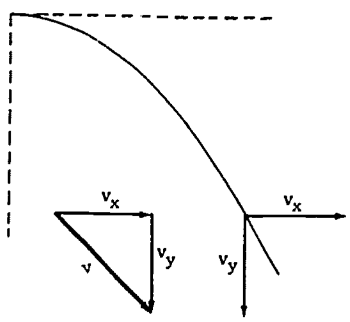
$$v_x = \text{constant}$$

$$x = v_x t$$

and for the vertical component of motion,

$$v_y = a_g t$$

$$y = \tfrac{1}{2} a_g t^2.$$



Because x, y, and d and v_x, v_y and v are the sides of right triangles, the magnitude d of the total vector displacement $\vec{d}$ can be written as

$$d = \sqrt{x^2 + y^2}$$

and the magnitude v of the velocity $\vec{v}$ can be written as

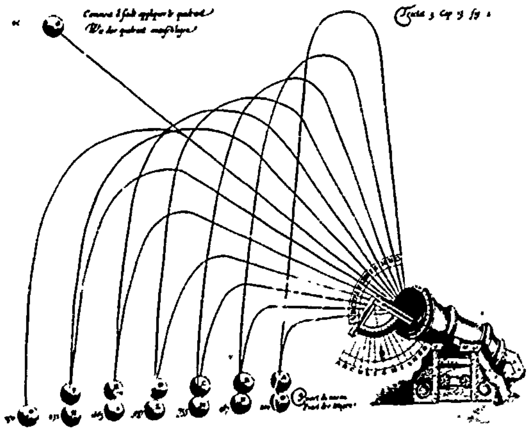
$$v = \sqrt{v_x^2 + v_y^2}\ .$$

See Study Guide 4.3.

Q1 A projectile is launched horizontally with a muzzle velocity of 1,000 m/sec from a point 20 m above the ground. How long will it be in flight? How far, horizontally, will the projectile travel?

4.3 What is the path of a projectile? It is easy to see that a thrown object, such as a rock, follows a curved path. But there are many kinds of curves, and it is not so easy to see which kind of curve a projectile traces. For example, arcs of circles, ellipses, parabolas, hyperbolas and cycloids (to name only a few geometric figures) all provide likely looking curved paths.

Ufano, a contemporary of Galileo, held a common belief about projectile trajectories. He thought that a projectile rises along a rather flat path, and then drops suddenly. Ufano was wrong, but more important is the fact that by direct observation of the moving object itself one could not determine the details of the trajectory.



The path taken by a cannon ball according to a drawing by Ufano (1621). He shows that the same horizontal distance can be obtained by two different firing angles. Gunners had previously found this by experience. What angles give the maximum range?

New knowledge about the path of a projectile was gained when the power of mathematics was applied to the problem. This was done by setting out to derive an equation that would express the shape of the path. Only a few steps are involved. First let us list equations we already know for a projectile launched horizontally.

$$x = v_x t$$

and

$$y = \frac{1}{2} a_g t^2 .$$

We would know the shape of the trajectory if we knew the height of the projectile above the ground for any horizontal distance from the launch point; that is, if we knew y for any value of x. We can find the height, y, for any horizontal distance, x, by combining our two equations in a way that eliminates the time variable. Solving the horizontal distance equation for t, we get

$$t = \frac{x}{v_x} .$$

Because t means the same in both equations we can substitute for t in the vertical distance equation to obtain

Specialized equations such as these are <u>not</u> to be memorized.

$$y = \tfrac{1}{2}a_g\left(\frac{x}{v_x}\right)^2.$$

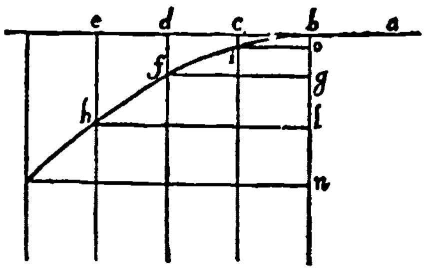
In this last equation there are two variables of interest, x and y, and three constant quantities: the number $\tfrac{1}{2}$, the uniform acceleration of free fall a_g, and the horizontal speed v_x which is constant for any one flight. Bringing these constants together, we can write the equation as

$$y = \left(\frac{a_g}{2v_x^2}\right) x^2$$

or

$$y = kx^2$$

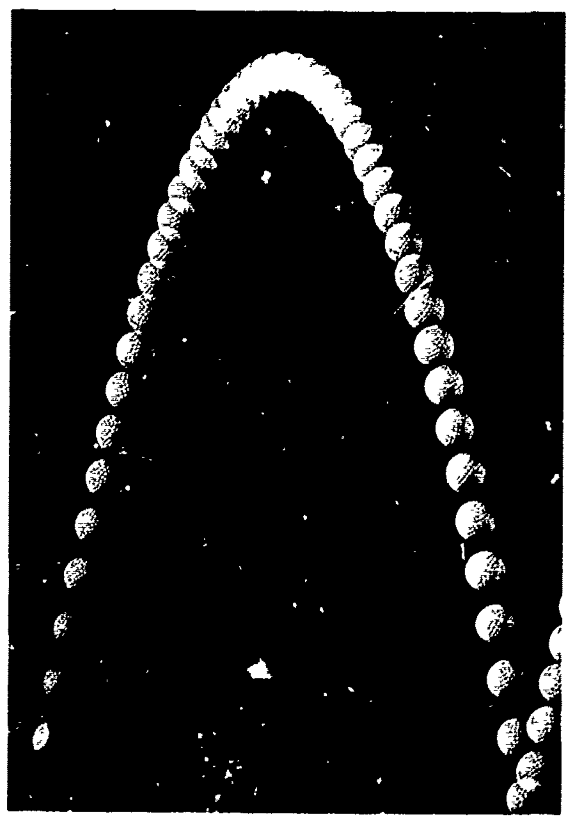
where $$k = \left(\frac{a_g}{2v_x^2}\right).$$



The parabolic path of a projectile fired horizontally to the left as deduced by Galileo on theoretical grounds in his Dialogues Concerning Two New Sciences. What is the relation between distances bo, og, and gl; and why?

This equation relates x and y for the trajectory. We can translate it as: the distance a projectile falls (vertically) is proportional to the square of how far it moves sideways (horizontally).

The mathematical curve represented by this relationship between x and y is called a parabola. Galileo deduced the parabolic nature of the trajectory (by an argument similar



to the one we used). With this discovery, the study of projectile motion became much simpler, because the geometric properties of the parabola had been established centuries earlier by Greek mathematicians.

Here we find a clue to one of the important strategies in modern science. When we express the features of a phenomenon quantitatively and cast the relations between them into equation form, we can use the rules of mathematics to manipulate the equations, and open the way to unexpected insights.

Galileo insisted that <u>the proper language of nature is mathematics</u>, and that an understanding of natural phenomena is aided by translating our qualitative experiences into quantitative terms. If, for example, we find that trajectories have a parabolic shape, we can apply all we know about the mathematics of parabolas to describe—and predict—trajectories. There is always a need for well-developed systems of pure mathematics which the physicist may use to express in precise form his conceptions of natural phenomena.

"Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it." (<u>Discoveries and Opinions of Galileo</u>, translated by Stillman Drake, Anchor Books, pp. 237-238.)

Moreover, the physical scientist often tries to use methods from another branch of science, or from mathematics, to find a solution for his particular problem. For example, just as Galileo applied the already known mathematics of parabolas to estimate actual projectile motions, so the modern sound engineer solves problems in acoustics using mathematical schemes developed independently by electrical engineers. Whatever the methods of science may be, many ideas and concepts can often be extended from one specialty to another with fruitful results.

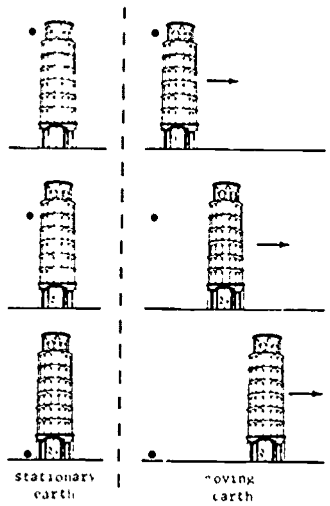
We can now apply our theory of projectile motion to the descent of a space capsule onto the moon's surface. The retro rockets of the orbiting capsule are fired to decrease its speed. After the retro rockets are turned off, the capsule's horizontal velocity (the velocity component parallel to the moon's surface) remains constant and the capsule falls freely under the influence of the moon's gravity. The path followed by the capsule with respect to the moon's surface is a parabola. Space-flight engineers are able to apply these ideas to land a space capsule on a desired moon target.

See Study Guide 4.4.

Q2 In the derivation of the path followed by a projectile, what assumptions have been made?

Write an equation for the trajectory of a projectile launched horizontally on the moon.

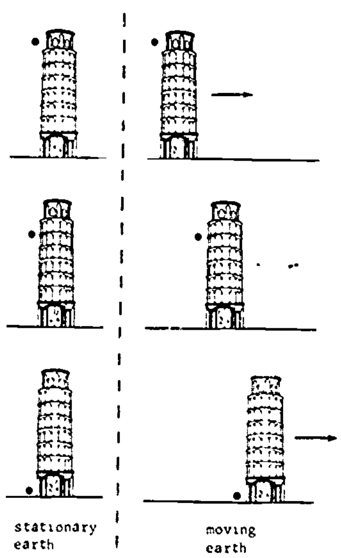
Q4 What is the constant in your equation in Q3?



The critics of Galileo claimed that if the earth moved, a dropped ball would land beyond the foot of the tower.

4.4 Galilean relativity. Galileo's work on projectiles illustrates the importance of reference frames. As you will see in Unit 2, Galileo ardently supported the idea that the reference frame for discussing motions in our planetary system be t ie ixed to the sun, not the earth, and that therefore the earth both rotates on its own axis and moves in a path around the sun. For many scientists of Galileo's time, this was not an easy idea to accept. If the earth moved, they said, a stone dropped from a tower would not land directly at its base. As earth moves through space, they argued, the stone would be left behind while falling through the air, and consequently would land far behind the base of the tower. But this is not what happens, so many of Galileo's critics believed that the tower and the earth cannot be considered to be in motion.

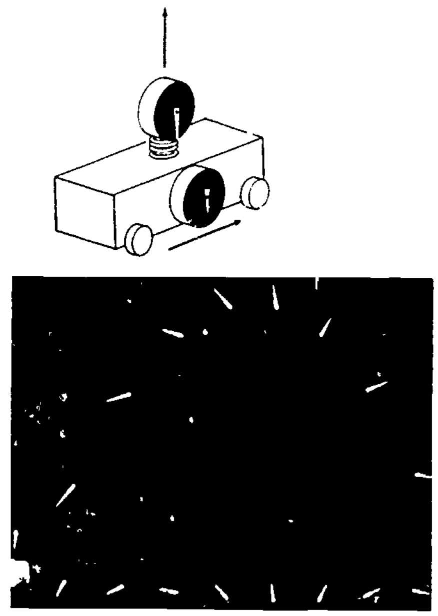
To answer these critics, Galileo first assumed that during the time of fall, the tower and the ground supporting it were moving forward equally with some uniform horizontal velocity v_x. He then claimed that the stone being held at the top of the tower also had the same horizontal velocity v_x, and that this velocity was not affected by the fact that the stone moves vertically upon being released. In other words, the falling stone behaves like any other projectile: the horizontal and vertical components of its motion are independent of each other. Since the stone and tower have the same v_x, the stone will not be left behind as it falls. Therefore, whether the speed of the earth is zero or not, the stone should land at the foot of the tower. So, the fact that falling stones are not "left behind" does not mean the earth is standing still.



Galileo argued that as the ball also shared the motion of the earth, an observer on earth could not tell whether or not the earth moved.

Similarly, Galileo said, an object released from a crow's nest at the top of a ship's straight mast will land at the foot of the mast whether the boat is standing still in the harbor or moving with constant velocity through quiet water. To a sailor standing on the ship, the trajectory will appear to be a straigh vertical line in either case. To a person standing on shore, however, the trajectory appears to be a straight vertical line when the ship is stationary, and a curved line when the ship is moving. Obviously, the frame of reference of the observer must be taken into account when analyzing the motion of objects. Galileo's explanation of the differing descriptions of the falling object was that the sailor on the deck of the moving ship is sharing the horizontal velocity of both the ship and the falling object. Sailor, ship, mast and object are all moving horizontally, so he cannot notice this component of the motion. By contrast, the observer on shore does not have the horizontal velocity v_x of the ship and object, and so he can see both the horizontal and vertical velocities of the falling object. These velocities, as we know, add vectorially to give a parabola.

The same ideas apply not only to falling bodies but also to projectiles in general. For example, if an object is projected vertically upward from a cart, it will fall back into the cart whether the cart is continuously moving at constant velocity or is standing still. From this, and equivalent observations, has come a most valuable generalization, usually called the Galilean relativity principle: any mechanical experiment will give the same result for any observer moving with constant velocity no matter what the magnitude and direction of the velocity. In other words, it is impossible to tell by any kind of mechanical experiment whether or not one's laboratory (reference frame) is really at rest or is moving with some constant velocity.



Two special clocks are attached to a cart. While the cart is moving at a constant speed, one of the clocks is sprung straight upwards from it and the subsequent motion of the two clocks is photographed under a stroboscopic light source. How do the horizontal positions of the two clocks compare in successive images?

From the Galilean relativity principle, it follows that the laws which describe mechanical experiments are the same in a reference frame at rest or in a reference frame moving with a constant velocity. Therefore, the laws for the description of the motion of projectiles would be found to be the same whether these laws are obtained by experiments inside a ship moving with constant velocity or at the dock; whether on a stationary earth or on an earth which, during any mechanical experiment on projectiles, is moving with virtually a constant velocity. In all these cases, we are in inertial frames of reference and we would arrive at a set of equations identical to the ones we have encountered in this and the earlier chapters.

The questions in Study Guide 4.5 and 4.6 deal with Galilean relativity.

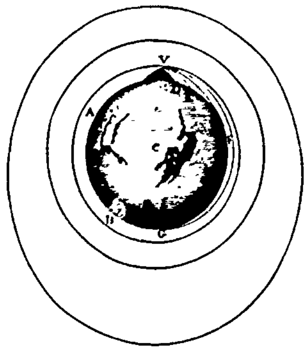
Before turning to circular motion, consider the famous "monkey in the tree" problem. It is described in Study Guide 4.7.

Q5 Compare the results of Galileo's inclined plane experiment performed in an elevator under the following circumstances:

a) elevator at rest.
b) elevator moving uniformly upward.
c) elevator moving uniformly downward.
d) elevator accelerating uniformly upward.
e) elevator accelerating uniformly downward.

Q6 For which experiment in Q5 would a_g appear to be the largest?

4.5 Circular motion. A projectile launched horizontally from a tall tower strikes the earth at a point determined by the speed of the projectile, the height of the tower and the acceleration due to the force of gravity. As the projectile's launch speed is increased, it strikes the earth at points farther and farther from the tower's base. (The assumptions we made in the analysis of projectile motion such as a "flat" earth which in turn implies a fixed direction of the gravitational force are no longer valid.) If we suppose the launch speed to be increased even more, the projectile would strike the earth at points ever farther from the tower, till at last it would rush around the earth in a near circular orbit. At this orbiting speed, the "projectile" is traveling so fast that its vertical fall just keeps pace with the receding surface of the curved earth.

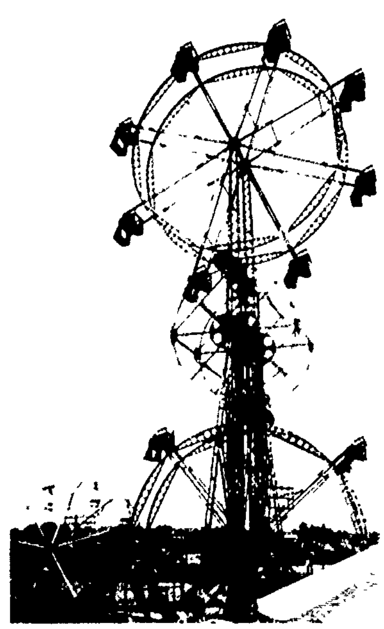


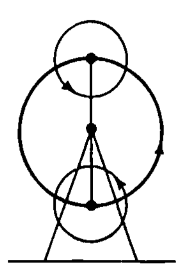
In discussing circular motion it is useful to keep clearly in mind a distinction between <u>revolution</u> and <u>rotation</u>. We define these terms differently: revolution is the act of traveling along a circular path; rotation is the act of spinning without traveling at all. A point on the rim of a phonograph turntable travels a long way; it is <u>revolving</u> about the axis of the turntable. But the turntable as a unit does not move from place to place: it merely <u>rotates</u>. In some situations both processes occur at once; for example, the earth rotates about its own axis while it also revolves (in a nearly circular path) around the sun.

What horizontal launch speed is required to put an object into a circular orbit about the earth? We shall be able to answer this question quite easily when we learn more about circular motion. We will also then be able to consider our problem of the capsule circling the moon.

The simplest kind of circular motion is uniform circular motion, that is, motion in a circle at constant speed. If you drive a car around a perfectly circular track so that at every instant the speedometer reading is forty miles per hour, you are executing uniform circular motion. But you will not be doing so if the track is any shape other than circular, or if your speed changes at any point.

How does one find whether an object in circular motion is moving at constant speed? The answer, surely, is to apply the same test we use in deciding whether or not an object traveling in a straight line does so with constant speed. We measure the instantaneous speed at many different moments and see whether the values are the same. If the speed is constant, we can describe the circular motion of an object by means of two numbers: the radius R of the circle and the speed v along the path. Instead of the speed, however, we shall





The circular motion of a double ferris wheel.

use a quantity easier to measure: either (1) the time required by an object to make one complete revolution, or (2) the number of revolutions the object completes in a stated interval of time. These latter two concepts have have been given names. The time required for an object to complete one revolution in a circular path is called the period of the motion. The period is denoted by the letter T. The number of revolutions completed by the same object in a specified time is called the frequency of the motion. Frequency will be denoted by the letter f.

Many commercial record turntables are designed to rotate at frequencies of 16 2/3 rpm (called transcription speed), 33 1/3 rpm (for LP's), 45 rpm (pop records), and 78 rpm (old-fashioned). What is the period corresponding to each of these frequencies?

In these terms we can describe a car moving with uniform speed on a circular track. Let us suppose the car takes 20 seconds to make one lap around the track. Thus, T = 20 seconds. Alternatively, we might say that the car makes 3 laps per minute, that is, 3/60 = 1/20 laps per second. Therefore, f = 1/20 revolutions/sec or more briefly f = 1/20 sec^{-1}. In this last expression the symbol sec^{-1} stands for 1/sec, or "per second." When the same time unit is used, the relationship between frequency and period is

$$f = \frac{1}{T}.$$

Any convenient units may be used. Radius may be expressed in terms of centimeters, kilometers, miles, or any other distance unit. Period may be expressed in seconds, minutes, or years. Correspondingly, the frequency may be expressed as "per second," "per minute," or "per year." The most widely used units of radius, period and frequency in scientific work are meter, second and per second.

Table 4.1

A comparison is shown below of the frequency and period for various kinds of circular motion. Note the changes in units. Can you put all the values in the table in seconds and per sec?

Phenomena	Period	Frequency
Electron in atom	10^{-16} sec	10^{16} per sec
Ultra-centrifuge	0.00033 sec	3000 per sec
Hoover Dam turbine	0.33 sec	3 per sec
Rotation of earth	24 hours	0.0007 per min
Moon around the earth	30 days	0.001375 per hour
Earth about the sun	365 days	0.0027 per day

If an object is in uniform circular motion, a person who knows the frequency of revolution f and the radius R of the path can compute the speed v of the object without difficulty.

The distance traveled in one revolution is simply the perimeter of the circular path, that is, $2\pi R$. The time for one revolution is just the period T. Thus since

$$\text{speed} = \frac{\text{distance traveled}}{\text{time elapsed}}$$

by substitution we can get

$$v = \frac{2\pi R}{T}\ .$$

To reformulate this circular motion equation in terms of frequency f we rewrite it as

$$v = (2\pi R)\ \frac{1}{T}\ .$$

Now since by definition

$$f = \frac{1}{T}\ ,$$

we find that

$$v = (2\pi R)(f) = 2\pi Rf.$$

If the body is in uniform circular motion, the speed computed with the aid of this equation is both the instantaneous speed and the average speed. If the motion is nonuniform, the formula gives only the average speed. The instantaneous speed can be determined only if we are able somehow to find $\Delta d/\Delta t$ from measurements of very small segments of the path.

Let us now see how this equation can be used. We can calculate the speed of the tip of a helicopter rotor blade as the helicopter sits on the ground. On one model, the main rotor has a diameter of 7.6 m and a frequency of 450 revolutions/minute under standard conditions. Thus f = 450 per minute and R = 3.8 m, so

$$v = 2\pi Rf$$

$$v = 2\pi(3.8)(450) \text{ meters/minute}$$

$$v = 10{,}700 \text{ meters/minute,}$$

See Study Guide 4.9 and 4.11.

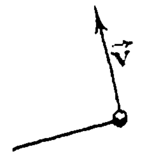
or about 400 mph.

Q7 A phonograph turntable makes 90 revolutions in 120 seconds.

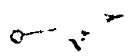
a) What is its period (in seconds)?
b) What is its period (in minutes)?
c) What is its frequency in cycles per second?

Q8 What is the period of the minute hand of an ordinary clock? If the hand is 6.0 cm long, what is the linear speed of the tip of the minute hand?

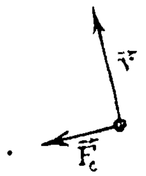
4.6 Centripetal acceleration. Let us assume that a stone, whirling on a string, is moving with uniform circular motion. The speed of the stone is constant. The velocity, however, is continuously changing because the direction of motion is continuously changing. At any instant, the direction of the velocity is tangent to the circular path. Since the velocity is changing, the stone is accelerating.



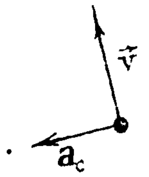
To keep the stone moving in a circular path, that is, to produce an acceleration, a force is needed. In the case of the whirling stone, a force is exerted on the stone by the string. If the string were suddenly cut, the stone would go flying off with the velocity it had at the instant the string was cut. As long as the string holds together, the stone is forced into a circular path.



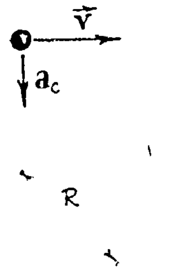
The direction of the force acting on the stone is along the string. Thus the force is always pointing toward the center of rotation. This kind of force—always directed toward the center of rotation—is called a centripetal force. (The adjective centripetal literally means "tending toward the center.") We shall give centripetal force the symbol $\vec{F}_c$. In uniform circular motion, the centripetal force always makes a right angle with the instantaneous velocity. As long as the force and the instantaneous velocity are at right angles, the magnitude of the velocity (that is the speed) does not change.



From Newton's second law we know that force and acceleration are in the same direction. Thus, the acceleration of the stone moving with constant speed along a circular path must, like the force, be directed toward the center of rotation. Furthermore, like the force, the acceleration always makes a right angle with the instantaneous velocity. We shall call this acceleration centripetal acceleration and give it the symbol $\vec{a}_c$. Any object moving along a circular path has a centripetal acceleration.



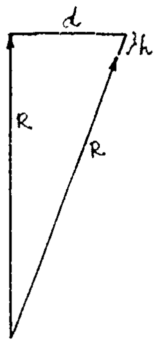
We know the direction of centripetal acceleration. What is its magnitude? We can determine the magnitude of the centripetal acceleration by an analysis of the figures on the next page. Assume the stone is moving in a circle of radius R. At any instant, the stone has a velocity, it has an acceleration, and it has a force exerted on it by the string. In order to keep the stone moving with constant speed in a circular path, a definite relationship between the magnitudes of the velocity, v, and the centripetal acceleration a_c, must exist. We can find what this relationship is by treating a small part of the circular path as the combination of a tangential motion and an acceleration toward the center. To follow the circular path, the stone must accelerate toward the center through a distance h <u>in the same time</u> that it would move a tangential distance d. The stone, with speed v, would travel a horizontal distance d given by $d = vt$. In the same time t, the stone, with acceleration a_c, would travel toward the center a distance h given by $h = \frac{1}{2}a_c t^2$. (We can use this last equation because at $t = 0$, the stone's velocity toward the center is zero.)



We can now apply the Pythagorean Theorem to the triangle in the figure above.

$$R^2 + d^2 = (R + h)^2 = R^2 + 2Rh + h^2.$$

When we cancel the like terms on each side of the equation, we are left with



$$d^2 = 2Rh + h^2.$$

We can simplify this expression by making an approximation: since h is very small compared to R, h^2 will be very small compared to Rh. So we shall neglect h^2 and write

$$d^2 = 2Rh.$$

However, we know $d = vt$ and $h = \frac{1}{2}a_c t^2$ so we can substitute for d^2 and for h. Thus

$$(vt)^2 = 2R\,(\tfrac{1}{2}a_c t^2)$$

$$v^2t^2 = Ra_c t^2$$

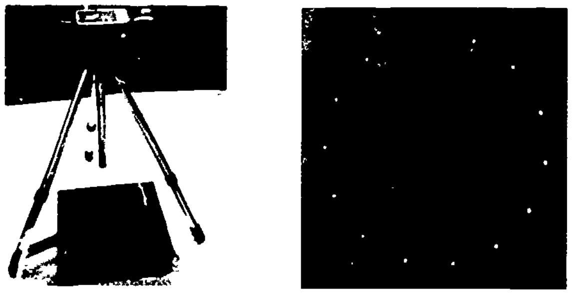
$$v^2 = Ra_c$$

or

$$a_c = \frac{v^2}{R}\ .$$

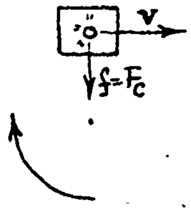
This is the magnitude of the centripetal acceleration for an object moving with a speed v on a circular path of radius R.

Let us verify this relationship. A photograph has been made of a blinky which was placed on a rotating phonograph turntable. The photograph and the actual setup are shown below. The blinky travels in a circular path with constant speed. The centripetal force in this case is the frictional force acting between the blinky and the surface of the phonograph turntable.

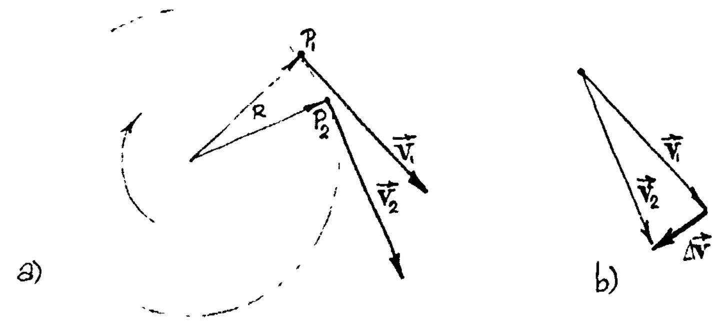


(a) The laboratory equipment for the rotating blinky experiment. (b) A photographic record of one revolution by the blinky. The blinky had a frequency of 9.4 sec and its path has a radius of 10.6 cm.

We shall determine the acceleration of the blinky by two methods. The first method makes use of the basic definition of acceleration, $\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$. The second makes use of the equation $a_c = \frac{v^2}{R}$.



As the blinky travels around its circular path, it may be at position P_1 at some instant and at position P_2 a short time later. At each such position its velocity can be represented by a vector. Since the circular motion is uniform, the arrows representing $\vec{v}_1$ and $\vec{v}_2$ must be equal in length. However, the vectors $\vec{v}_1$ and $\vec{v}_2$ differ in direction. What is the difference between the vectors? The figure below shows the two vectors (arranged with the tails at the



same point, to make the comparison easier) and makes clear that they differ by the vector labeled $\Delta\vec{v}$. That is, $\Delta\vec{v}$ has a direction and a magnitude such that

$$\vec{v}_1 + \Delta\vec{v} = \vec{v}_2.$$

By rearrangement, this becomes

$$\Delta v = \vec{v}_2 - \vec{v}_1,$$

which is the definition of "change in velocity."

In words, this equation means that in the short time interval Δt in which the blinky travels from P_1 to P_2, it must acquire a new component of velocity—a component having the direction and magnitude of $\Delta\vec{v}$. The direction of $\Delta\vec{v}$ is toward the center of the circle.

From measurements on the revolving blinky, we have determined the speed v. Using an appropriate scale factor (in this case 1.0 cm stands for 45 cm/sec) we plotted the velocity at points P_1 and P_2 and determined Δv by a direct length measurement. The magnitude of the change in velocity was found to be 20 cm/sec. The rest is straight calculation. The time interval Δt between flashes was .11 sec, and therefore:

Can you determine the speed of the blinky from the data given?

$$\text{the magnitude of the acceleration} = \frac{\text{the magnitude of the change in velocity}}{\text{the change in time}}$$

$$= \frac{20 \text{ cm/sec}}{.11 \text{ sec}}$$

$$= 190 \text{ cm/sec}^2.$$

How is Δt = .11 sec obtained from the information that f = 9.4 flashes/sec?

Thus, by a combination of graphical and algebraic steps, we found the magnitude of the acceleration the blinky underwent as it revolved on the turntable.

There were 9.4 blinks/sec and a total of 14 blinks; therefore the period T must be

$$\frac{14 \text{ blinks}}{9.4 \text{ blinks/sec}},$$

or

$$T = 1.5 \text{ sec}.$$

Let us now use the equation, $a_c = \frac{v^2}{R}$, to find the centripetal acceleration of the blinky and compare it to the results obtained by the graphical method. The information we have is R = 10.6 cm and T = 1.5 sec. The speed of the blinky is given by

$$v = \frac{2\pi R}{T}.$$

Substituting in numerical values we get

$$v = \frac{2(3.14)(10.6) \text{ cm}}{1.5 \text{ sec}}$$

$$= 44 \text{ cm/sec}.$$

For this and most other problems on uniform circular motion, it is only necessary to remember and understand

$$v = \frac{2\pi R}{T},$$

$$f = \frac{1}{T}, \text{ and}$$

$$a_c = \frac{v^2}{R}.$$

We can substitute this value for speed into the expression we derived for the acceleration.

$$a_c = \frac{v^2}{R}$$

$$= \frac{(44.4 \text{ cm/sec})^2}{10.6 \text{ cm}}$$

$$= \frac{1971 \text{ cm}^2/\text{sec}^2}{10.6 \text{ cm}}$$

$$= 190 \text{ cm/sec}^2.$$

The answers obtained by the two methods agree.

If v^2/R is the magnitude of the centripetal acceleration, then from Newton's second law we can conclude that mv^2/R is the magnitude of the centripetal force. The hammer thrower in the photograph is exerting a tremendous centripetal force to keep the hammer moving in a circle as he speeds it up. From the distance the hammer travels, we can estimate its speed at release. To keep the 16-pound hammer in a circle at the release speed requires over 500 pounds of force!

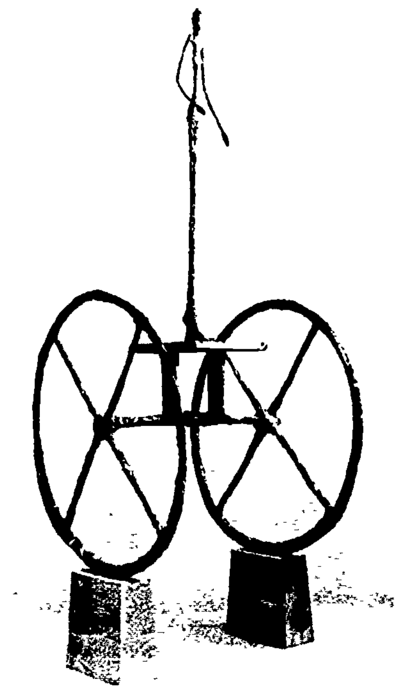
Let us return to our space flight. The space capsule in Part 5 of our earth-moon flight is orbiting the moon in a circle at a constant speed. From the radius of the orbit and the capsule's speed, we can compute the centripetal acceleration and, if we know the capsule's mass, the centripetal force. What is the origin of the centripetal force? If you do not already know, you will find out in Unit 2. By knowing what the centripetal force is, space engineers can work the problem backwards to determine the speed the capsule must have for a particular lunar orbit.

See Study Guide 4.12, 4.14, and 4.15 for further thoughts on centripetal acceleration.

Q9 In the last section we calculated that the tip of a helicopter rotor blade ($f = 450\ min^{-1}$ and $R = 3.8$ m) was moving about 10,700 m/sec. Find centripetal acceleration of the tip.

4.7 The motion of earth satellites. Nature and technology provide many examples of the type of motion where an object is in uniform circular motion. The wheel has been a main characteristic of our civilization, first as it appeared on crude carts and then later as an essential part of complex machines. The historical importance of rotary motion in the development of modern technology has been described by the historian V. Gordon Childe:

> Rotating machines for performing repetitive operations, driven by water, by thermal power, or by electrical energy, were the most decisive factors of the industrial revolution, and, from the first steamship till the invention of the jet plane, it is the application of rotary motion to transport that has revolutionized communications. The use of rotary machines, as of any other human tools, has been cumulative and progressive. The inventors of the eighteenth and nineteenth centuries were merely extending the applications of rotary motion that had been devised in previous generations, reaching back thousands of years into the prehistoric past.... [V. Gordon Childe "Rotary Motion" in The History of Technology, ed. Charles Singer, E. J. Holmyard, and A. R. Hall, Vol. I (New York: Oxford University Press, 1953) p. 187.]

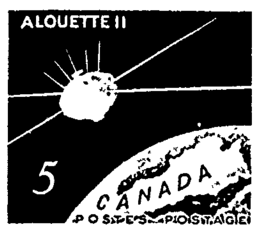


Chariot. Alberto Giacometti, 1950. Courtesy Museum of Modern Art.

We shall see in Unit 2 that another rotational motion, that of the orbiting planets around the sun, has also been one of the central concerns of man throughout recorded history.

Since the kinematics and dynamics of all uniform circular motion are the same, we can apply what we have learned to the motion of artificial earth satellites in circular (or nearly circular) paths. The satellite selected for study here is Alouette, Canada's first satellite, which was launched into a nearly circular orbit by a Thor-Agena B rocket on September 29, 1962.

Alouette is orbiting at an average distance of 4,593 miles from the center of the earth. Its closest approach to the earth is 620 miles, and its farthest distance from the earth



Alouette I still provides useful data upon command. Alouette II was placed in orbit in late 1965.

Table 4.2 Some information on selected artificial earth satellites.

Name	Launch date	Weight (lb)	Period (min)	Height (miles) Perigee-Apogee	Remarks
Sputnik 1 1957α (USSR)	Oct. 4, 1957	184	96.2	142-588	First earth satellite. Internal temp., pressure.
Explorer 7 1958α (USA)	Jan. 31, 1958	30.8	114.8	224-1573	Cosmic rays, micrometeorites, internal and shell temps., discovery of first Van Allen belts.
Lunik 3 1959θ (USSR)	Oct. 4, 1959	959	22,300	30,000-291,000	Transmitted photographs of far side of moon.
Vostok 1 1961μ (USSR)	Apr. 12, 1961	10,416	89.34	109-188	First manned orbital flight (Major Yuri Gagarin; one orbit).
Midas 3 1961σ (USA)	July 12, 1961	3,500	161.5	2,129-2,153	Almost circular orbit.
Telstar 1 1962αε (USA)	July 10, 1962	170	157.8	593-3,503	Successful transmission across the Atlantic: telephony, phototelegraphy, and color and black and white television.
Alouette 1 1962βα (USA-CANADA)	Sept. 29, 1962	319	105.4	620-640	Joint project between NASA and Canadian Defense Research Board; measurements in ionosphere.
Luna 4 1963-08 (USSR)	Apr. 2, 1963	3,135	42,000	56,000-435,000	Passed 5,300 miles from moon; very large orbit.
Vostok 6 1963-23 (USSR)	June 16, 1963	"about 5 tons"	88.34	106-134	First orbital flight by a woman; (Valentina Terishkova; 48 orbits).
Syncom 2 1963-31 (USA)	July 26, 1963	86	1,460.4	22,187-22,192	Successfully placed in near-synchronous orbit.

is 640 miles. Since this path is so nearly circular, we will treat it as a circle in our analysis of the satellite's motion. The speed of Alouette can be taken to be constant for our purposes, since it varies less than one mile per minute above or below the average speed of 275 miles per minute.

Now let us compute the orbital speed and centripetal acceleration of Alouette. The relationship $v = 2\pi R/T$ allows us to find the speed of any object moving uniformly in a circle if R and T are known. To determine a satellite's speed, we need to know its distance R from the center of the earth and its period T.

Tracking stations located in many places around the world maintain a record of any satellite's position in space. From the position data, the satellite's distance above the earth at any time and its period of revolution are found. By means of such tracking, we know that Alouette moves at an average height of 630 miles above sea level, and takes 105.4 min to complete one circular orbit. Adding 630 miles to the earth's radius, 3,963 miles, we obtain R = 4,593 miles, and

$$v = \frac{2\pi R}{T} \approx \frac{2\pi(4{,}590)\ \text{mi}}{105\ \text{min}} = \frac{28{,}800\ \text{mi}}{105\ \text{min}} = 275\ \text{mi/min}.$$

This is equivalent to 16,500 mi/hr, or 7,150 m/sec.

The last equation can be used to find the speed of any satellite, for example, that of our moon. The average distance from the center of the earth to the center of the moon is approximately 2.39×10^5 mi, and the moon takes an average of 27 days, 7 hrs, 43 min to complete one revolution around the earth with respect to the fixed stars. Thus

$$v = \frac{2\pi(2.39 \times 10^5)\ \text{mi}}{3.93 \times 10^4\ \text{min}} = 38.1\ \text{mi/min},$$

or roughly 2,280 mi/hr.

If we wish to calculate the centripetal acceleration of Alouette, we can use the value of v found above along with the relationship $a = \frac{v^2}{R}$. Thus

$$a = \frac{(275\ \text{mi/min})^2}{4{,}590\ \text{mi}} = 16.5\ \text{mi/min}^2$$

This is the equivalent of 7.42 m/sec^2. What force gives rise to this acceleration? (Hint: the acceleration of a falling stone at the surface of the earth is 9.80 m/sec^2.)

Earlier we asked the question, "What horizontal launch speed is required to put an object into a circular orbit about the earth?" Can you answer this question now? If not, turn to Study Guide 4.23 for help.

In Chapter 2 we found that the acceleration due to gravity at the earth's surface was about 9.8 m/sec^2. Here we have just calculated the acceleration of Alouette toward the center of the earth to be about 7.4 m/sec^2. Calculate the moon's centripetal acceleration.

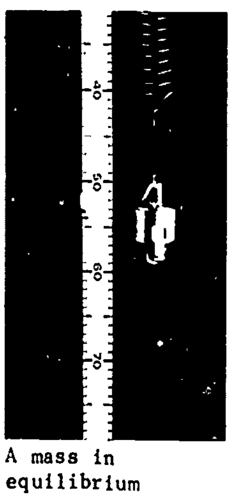
A vibrating string

Swinging children

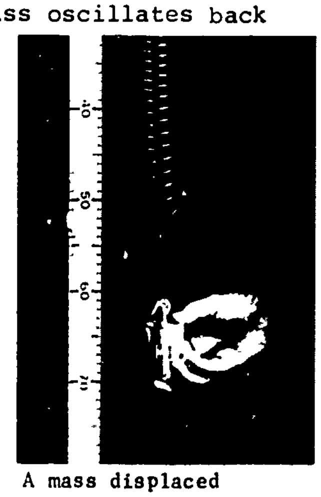
Simple Harmonic Motion

Back-and-forth motions similar to the swinging child and the vibrating guitar string are common. There are rocking boats, swaying trees, and vibrating tuning forks. There are clock pendula and quivering diving boards. What are the details of such oscillatory motions? What kind of force is acting on an oscillating object? No new concepts are needed to answer these questions, so let us proceed.

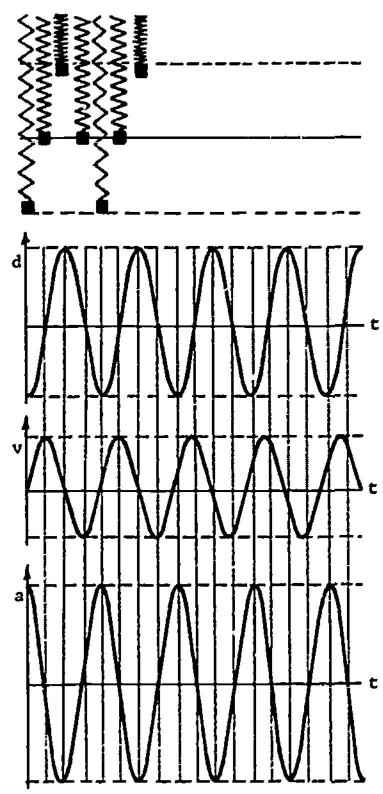
The mass on the spring pictured below is in equilibrium. If we displace the mass vertically from its equilibrium position, a force is exerted on the mass by the spring. This force, F_s, tends to restore the mass to its equilibrium position. Let us displace the mass, release it and observe its motion. We observe that the mass oscillates back and forth through its equilibrium position. If we start a timer at the instant of release, we could represent the displacement of the mass at any time on a graph. The displacement of the mass ranges from a maximum in one direction to a maximum in the other direction; that is, from + d to -d.



A mass in equilibrium



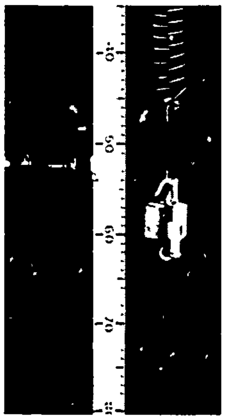
A mass displaced from equilibrium



As the mass approaches its maximum displacement, it slows down, stops and then speeds up in the opposite direction. The speed of the mass is the greatest as it passes through the equilibrium position. This information can also be represented graphically. The displacement-time, velocity-time and acceleration-time graphs are shown below.

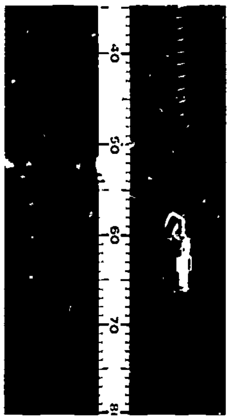
These graphs give us the kinematic details of the motion. From the graphs we see that the velocity is a maximum when the displacement is a minimum. Further, we see that when the displacement is a maximum in one direction, the acceleration is a maximum in the other direction.

What about the force exerted on the mass by the spring? By combining the information in the acceleration-time graph with Newton's second law, we know that the force is varying in both magnitude and direction. We can determine how the force varies by an experiment shown in the photographs below. In this experiment forces of known magnitudes—0.5 N and 1.0 N—were applied to the mass. From the photographs we can measure the displacements of the mass



F_s=0.5 N

d=3.7 cm



F_s=1.0 N

d=7.5 cm

resulting from the known forces. This measurement tells us the force that the spring is exerting on the mass at these two displacements.

That is, when

d = 3.7 cm then F_s = 0.5 N

and when

d = 7.5 cm then F_s = 1.0 N.

A close look at these results seems to indicate that $F_s \propto d$ or $F_s = kd$ where k is a constant of proportionality. (Verify for yourself that F_s is proportional to d. Remember Study Guide 3.9.)

One additional piece of information is needed before we fully understand the spring force F_s. What is the relation between the directions of $\vec{F}_s$ and $\vec{d}$? When the displacement is in the downward direction, the spring force is in the upward direction and vice versa. In other words, the force $\vec{F}_s$ is in the opposite direction of $\vec{d}$. We can now write the force law which expresses the nature of the force exerted by the spring on the mass. This force law is

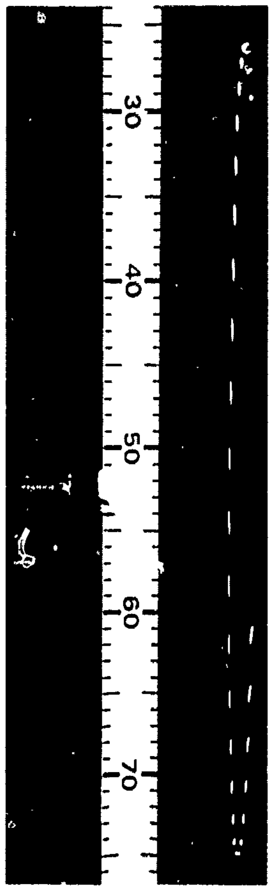
$$\vec{F}_s = -k\vec{d}.$$

The minus sign indicates the opposite directions of $\vec{F}_s$ and $\vec{d}$.

The back-and-forth motion resulting from the force $\vec{F}_s = -k\vec{d}$ is called simple harmonic motion. Are the swinging child and the vibrating guitar string examples of simple harmonic motion? Are they examples of motion for which the motivating force is proportional to the displacement? The answer to both of these questions is "only approximately." There are other forces which tend to slow down and bring to rest the swinging child and the guitar string. In other words, the back-and-forth motion is damped. If the damping forces are not large, these motions and many others besides closely approximate simple harmonic motion.

SUGGESTED ACTIVITY

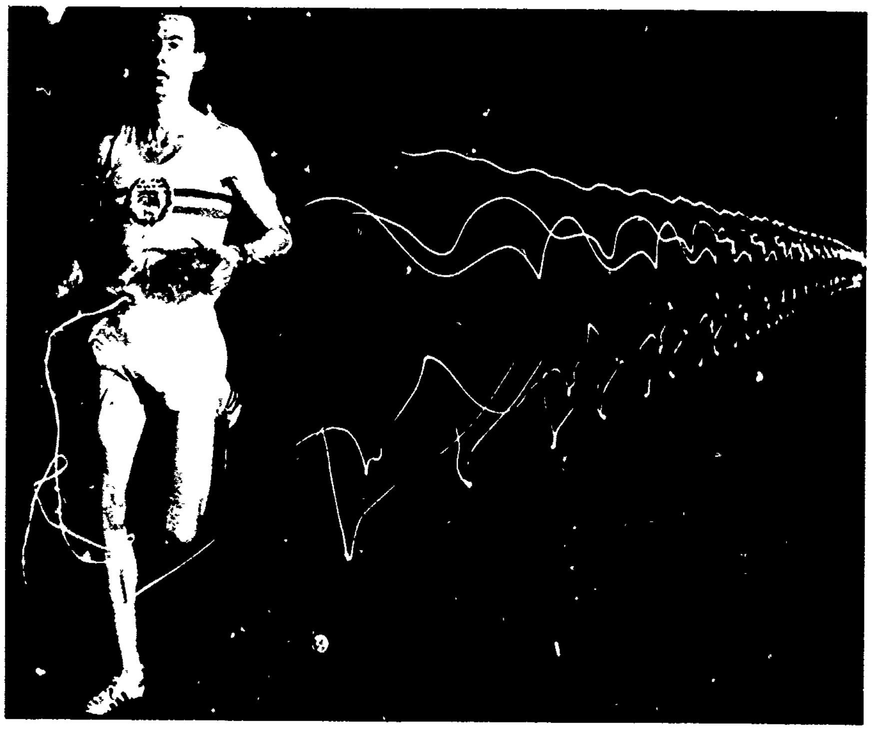
The stroboscopic photograph at the right shows the position of a light attached to the mass at time intervals of 1/30 second. The mass is 0.52 kg.

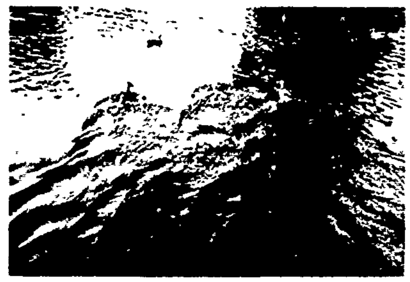


1. What is the equilibrium position?
2. Construct a displacement-time graph.
3. Measure the slope of the displacement-time graph at several different times and construct a velocity-time graph.
4. Determine the acceleration of the mass when it is positioned half-way between the maximum displacement and the equilibrium position.
5. What is the force exerted on the mass by the spring at the same point chosen in (4) above?
6. Does Newton's second law hold?
7. For additional suggested activities see Study Guide 4.22.

4.8 What about other motions? So far we have described rectilinear motion, both uniform and accelerated, projectile motion, uniform circular motion, and simple harmonic motion. Taken together, these descriptions are useful in clarifying much of interest in the world of motion. Even so, it is clear that we still have avoided many complicated kinds of motion that may interest us. For instance, consider these:

a) the motion in a pattern of water ripples;
b) the motion of the Empire State Building;
c) the motion of a small dust particle as it zig-zags through still air;
d) a person running.





Even if we have not treated these motions directly, what we have done so far is of real value. The methods for dealing with motion which we have developed in this and the preceding chapters are important because they give us means for dealing with any kind of motion whatsoever. All motion can be analyzed in terms of position, velocity, and acceleration.

When we considered the forces needed to produce motion, Newton's laws supplied us with concise yet very general answers. Later, when we discuss the elliptical motion of planets, and the hyperbolic motion of an alpha particle passing near a nucleus, we shall be able to infer the magnitude and direction of the forces acting in each case.

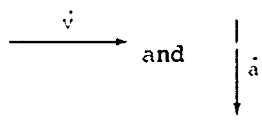
On the other hand, when we know the magnitude and direction of the force acting on an object, we can determine what its change in motion will be. If in addition to this, we know the position and velocity of an object, we can reconstruct how it moved in the past and we can predict how it will move in the future. Thus, Newton's laws provide a comprehensive view of forces and motion. It is not surprising that Newton's work was greeted with astonished wonder. Such wonder is aptly expressed in Alexander Pope's oft-quoted couplet:

> Nature and Nature's laws lay hid in night,
> God said, "Let Newton be!" and all was light.

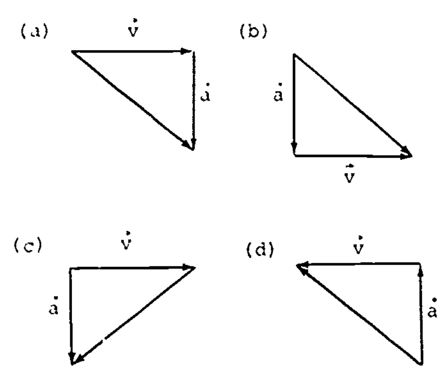
4.1 Using symbols other than words, give an example of each of these:

a) a scalar.
b) a vector.
c) the addition of two scalars.
d) the addition of two vectors.
e) the addition of three vectors.
f) the subtraction of one scalar from another.
g) the subtraction of one vector from another.

4.2 For a given moving object the velocity and acceleration can be represented by these vectors:

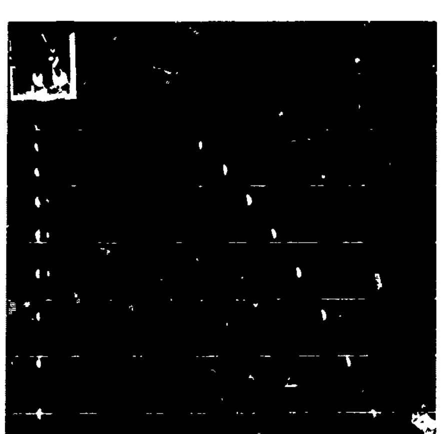


The sum of these two vectors is:



e) They cannot be added.

4.3 A sphere is launched horizontally, as shown below. Suppose the initial speed v_x is 3.0 m/sec. Where is the projectile (displacement), and what is its speed and direction (velocity) 0.5 sec after launching?



4.4 If a raindrop accelerated at a constant rate of 9.8 m/sec^2 from a cloud 1 mile up what would be its speed just before striking the ground. Does a raindrop accelerate at a constant rate over a 1 mile fall?

4.5 An airplane has a gun that fires bullets at the speed of 600 mph when tested on the ground with the plane stationary. The plane takes off and flies due east at 600 mph. Which of the following claims are correct, if any? In defending your answers, refer to Galilean relativity.

a) When fired directly ahead the bullets move eastward at a speed of 1200 mph.
b) When fired in the opposite direction, the bullets drop vertically downward.
c) If fired vertically downward, the bullets move eastward at 600 mph.

4.6 Two persons watch the same object move. One says it accelerates straight downward, but the other claims it falls along a curved path. Invent a situation in which they both could be right.

4.7 A hunter points his gun barrel directly at a monkey in a distant palm tree. Where will the bullet go? If the animal, startled by the flash, drops out of the branches at the very instant of firing, will it then be hit by the bullet? Explain.

4.8 If a broad jumper takes off with a speed of 10 m/sec at an angle of 45° with respect to the earth's surface, how far would he leap? If he took off from the moon's surface with that same speed and angle, what would be the length of his leap. The gravitational acceleration of a body at the moon's surface is $\frac{1}{6}$th of that at the earth's surface.

4.9 Contrast rectilinear motion, projectile motion, and uniform circular motion by:

a) defining each.
b) giving examples.
c) comparing the velocity-acceleration relationships.

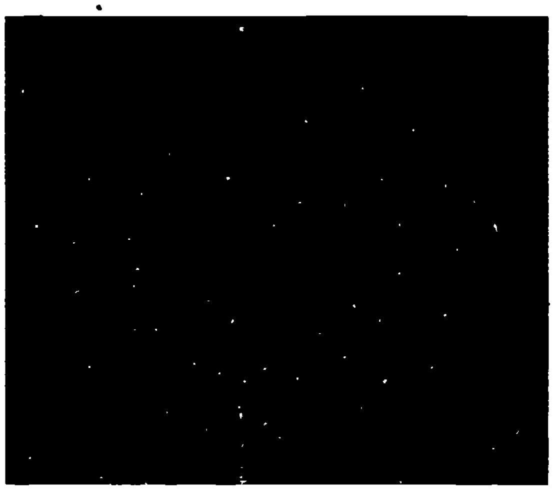
4.10 You are inside a uniformly accelerating moving van. If when the van is traveling at 10 mph (and still accelerating) you dropped a ball from the roof of the van onto the floor, what would be the ball's path relative to the van? What would be its path relative to a person driving past the van in the opposite direction of the van at a uniform speed? What would be its path relative to a person standing on road?

4.11 An object in uniform circular motion makes 20 revolutions in 4.0 sec.

a) What is its period T?
b) What is its frequency f?
c) If the radius of rotation is 2 meters, what is its speed?

4.12 Two blinkies were placed on a rotating turntable and photographed from directly overhead. The result is shown in the figure below. The outer blinky has a frequency of 9.4 flashes/sec and is located 15.0 cm from the center. For the inner blinky, the values are 9.1 flashes/sec and 10.6 cm.

a) What is the period of the turntable?
b) What is the frequency of rotation of the turntable? Is this a standard phonograph speed?
c) What is the linear speed of the turntable at the position of the outer blinky?
d) What is the linear speed of the turntable at the position of the inner blinky?
e) What is the linear speed of the turntable at the center?
f) What is the angular speed of each blinky in degrees/sec? Are they equal?
g) What is the centripetal acceleration experienced by the inner blinky?
h) What is the centripetal acceleration experienced by the outer blinky?



4.13 These questions are asked with reference to Table 4.2 on page 112.

a) Are the distances to apogee and perigee given as height above the surface of the earth or distance from the center of the earth?
b) Which satellite has the most nearly circular orbit?
c) Which are the most eccentric? How did you arrive at your answer?
d) Which satellite in the table has the longest period?
e) What is the period of Syncom 2 in hours.
f) How does the position of Syncom relative to a point on the earth change over one day.
g) Which satellite has the greater centripetal acceleration, Midas 3 or Syncom 2?
h) What is the magnitude of the centripetal acceleration of Vostok 6. Express answer in m/sec^2.

4.14 The following table shows the period and the mean distance from the sun for the three planets that most nearly go in a circular orbit.

Planet	Mean distance (r) from sun (in A.U.)	Period (T) in years
Venus	0.72	0.62
Earth	1.00	1.00
Neptune	30.06	164.8

(A.U. = astronomical unit = the mean distance of the earth from the sun; 1 A.U. = 92.9×10^6 miles.)

a) What is the average orbital speed for each planet (in A.U./year)?
b) Calculate the centripetal acceleration for each planet in $A.U./yr^2$.
c) Can you see any relationship between the mean distance and the centripetal acceleration a_c?

[Hint: Does it appear to be

(1) $a_c \propto r$, or (2) $a_c \propto 1/r$; or

(3) $a_c \propto r^2$; or (4) $a_c \propto 1/r^2$?

How can a graph help you to decide?]

4.15 Explain why it is impossible to have an earth satellite orbit the earth in 80 minutes. Does this mean that it is impossible for an object to circle the earth in less than 80 minutes?

4.16 The intention of the first four chapters has been to describe "simple" motions and to progress to the description of "complex" motions. Organize the following examples into a list from the simplest to the most complex, making whatever idealizing assumptions you wish. Be prepared to say why you placed any one example ahead of those below it, and to state any assumptions you made.

A "human cannon ball" in flight
A car going from 40 mph to a complete stop
A redwood tree
A child riding a ferris wheel
A rock dropped 3 m
A woman standing on an escalator
A climber ascending Mt. Everest

4.17 Could you rank the above examples if you were not permitted to idealize? If yes, how would you then rank them? If no, why not?

4.20 Compare the centripetal acceleration of the tire tread of a motor scooter wheel (diameter 1 ft) with that of a motorcycle wheel (diameter 2 ft) if both vehicles are moving at the same speed.

4.21 Our sun is located at a point in our galaxy about 30,000 light years (1 light year = 9.46×10^{12} km) from the galactic center. It is thought to be revolving around the center at a linear speed of approximately 250 km/sec. a) What is the sun's centripetal acceleration with respect to the center of our galaxy? b) If the sun's mass is taken to be 1.98×10^{30} kg, what centripetal force is required to keep it moving in a circular orbit about the galactic center? c) Compare the centripetal force in b) with that necessary to keep the earth in orbit about the sun. (The earth's mass is 5.98×10^{24} kg and its average distance from the sun is 1.495×10^{8} km. What is its linear speed in orbit?)

4.18 Using a full sheet of paper, make and complete a table like the one below.

Concept	Symbol	Definition	Example
		Length of a path between any two points as measured along the path	
			Straight line distance and direction from Detroit to Chicago
speed			
	$\bar{v}$		
			An airplane flying west at 400 mph at constant altitude
		Time rate of change of velocity	
	a_g		
Centripetal acceleration			
			The drive shaft of some automobiles turns 600 rpm in low gear
		The time it takes to make one complete revolution	

4.19 The diameter of the main wheel tires on a Boeing 727 fan jet is 1.26 m. The nose wheel tire has a diameter of 0.81 m. The speed of the plane just before it clears the runway is 86.1 m/sec. At this instant, find the centripetal acceleration of the tire tread, for each tire.

4.22 Here are a list of some possible investigations into simple harmonic motion.

1. How does the period of a pendulum depend upon

   a) the mass of the pendulum bob?
   b) the length of the pendulum?
   c) the amplitude of the swing (for a fixed length and fixed mass)?

2. How does the period of an object on the end of a spring depend upon

   a) the mass of the object?
   b) the spring constant, k, where the spring constant k is defined as the slope of the graph of force versus spring extension? Its units are newtons/meter.

4.23 The centripetal acceleration experienced by a satellite orbiting at the earth's surface (air resistance conveniently neglected) is the acceleration due to gravity of an object at the earth's surface (9.8 m/sec^2). Therefore, the speed required to maintain the satellite in a circular orbit must be such that the centripetal acceleration of the satellite is 9.8 m/sec^2. This condition can be expressed as follows.

$$a_c = \frac{V^2}{R} = a_g = 9.8 \text{ m/sec}^2$$

R, the radius of the earth, is 6.38×10^6 meters

$$a_g = 9.8 \text{ m/sec}^2$$

$$V^2 = 9.8 \text{ m/sec}^2 \times 6.38 \times 10^6 \text{ m}$$

$$= 62.5 \times 10^6 \text{ m}^2/\text{sec}^2$$

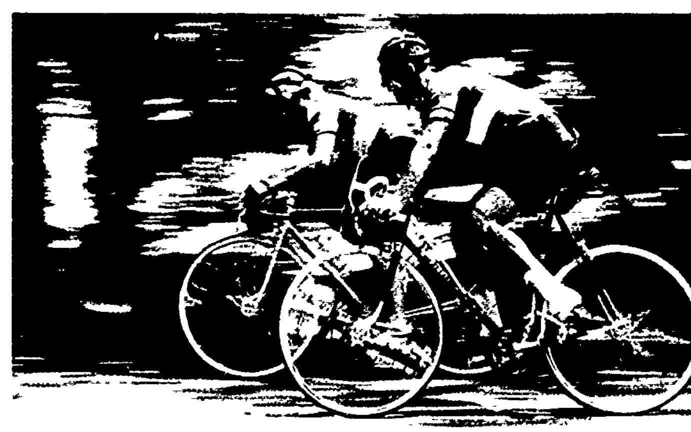
$$V = 7.85 \times 10^3 \text{ m/sec}$$

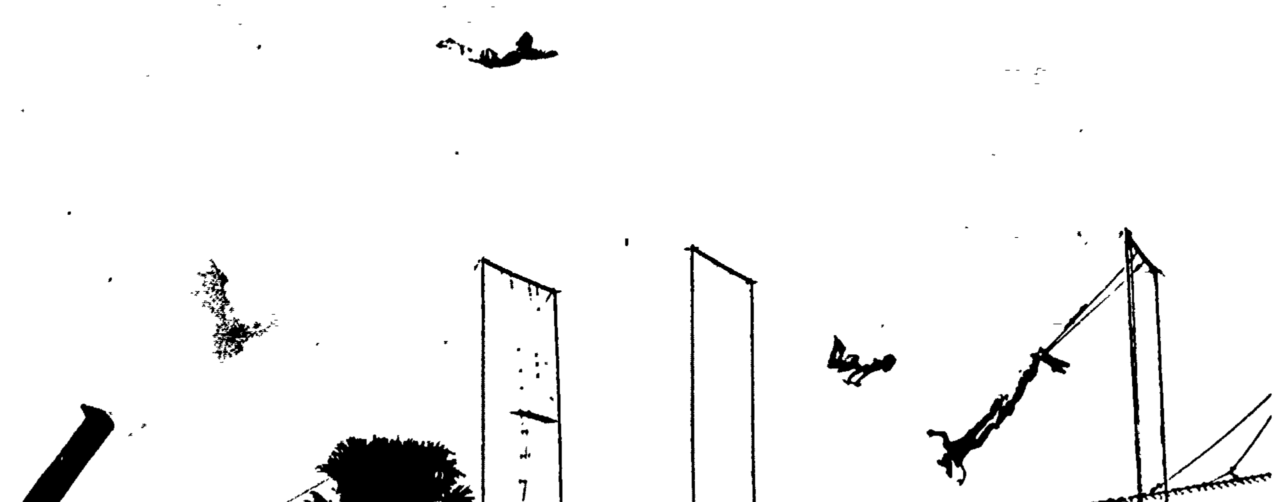
What is the period T of this orbit?

What is the satellite's speed expressed in miles per hour? (Hint: 1,000 meters = .61 miles.)

4.24 The thrust of a Saturn Apollo launch vehicle is 7,370,000 newtons (approximately 1,650,000 lbs) and its mass is 540,000 kg. What would be the acceleration of the vehicle relative to the earth's surface at lift off? How long would it take for the vehicle to rise 50 meters? The acceleration of the vehicle increases greatly with time (it is 47 m/sec^2 at first stage burnout), even though the thrust force does not increase appreciably. Explain why the acceleration increases.

4.25 Write a short essay on one of the following pictures.





Epilogue The purpose of this Unit was to deal with the fundamental concepts of motion. We decided to start by analyzing particularly simple kinds of motion in the expectation that we might discover the "ABC's" of physics. With these basic ideas it was hoped we could turn our attention back to some of the more complex (and more interesting) features of the world. To what extent were these expectations fulfilled?

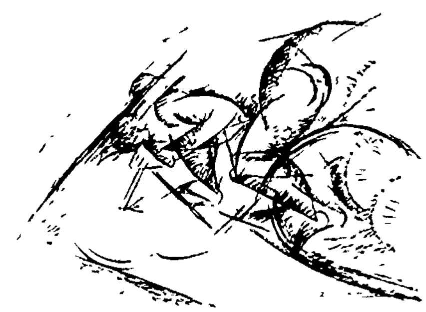
We did find that a relatively few basic concepts allowed us to gain a considerable understanding of motion. First of all, we found that useful descriptions of the motion of objects can be given using the concepts of distance, displacement, time, speed, velocity and acceleration. If to these we add force and mass and the relationships expressed in Newton's three laws of motion, it becomes possible to account for observed motion in an effective way. The surprising thing is that these concepts of motion, which were developed in extraordinarily restricted circumstances, can in fact be so widely applied. For example, our work in the laboratory centered around the use of sliding dry ice pucks and steel balls rolling down inclined planes. These are not objects to be found in the everyday "natural" world. Even so, we found that the ideas obtained from those experiments could be used to deepen our understanding of objects falling near the earth's surface, of projectiles, and of objects moving in circular paths. We started by analyzing the motion of a piece of dry ice moving across a smooth surface and ended up analyzing the motion of a space capsule as it circles the moon and crashes to its surface.

In other words, we really have made substantial progress. On the other hand, we cannot be satisfied that we have all of the intellectual tools necessary to understand all of the phenomena that interest us. We will find this to be especially true as we turn our attention away from interactions involving a relatively few objects of easily discernable size, and to interactions involving countless numbers of submicroscopic objects, i.e., molecules and atoms. Thus in Unit 3 we shall add to our stock of fundamental concepts a few additional ones, particularly those of momentum, work and energy.

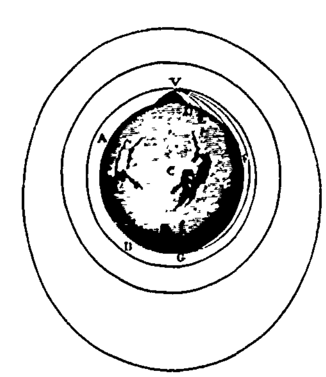
In this Unit we have dealt primarily with concepts that owe their greatest debts to Galileo, Newton and their followers. If space had permitted, we should also have included the contributions of René Descartes and the Dutch scientist, Christian Huyghens. The mathematician and philosopher,

A. N. Whitehead has summarized the role of these four men and the significance of the concepts we have been dealing with in this Unit in the following words:

> This subject of the formation of the three laws of motion and of the law of gravitation [which we shall take up in Unit 2] deserves critical attention. The whole development of thought occupied exactly two generations. It commenced with Galileo and ended with Newton's Principia; and Newton was born in the year that Galileo died. Also the lives of Descartes and Huyghens fall within the period occupied by these great terminal figures. The issue of the combined labours of these four men has some right to be considered as the greatest single intellectual success which mankind has achieved. [Science and the Modern World]



The revolution Whitehead speaks of, and the subject of this Unit, was important for many reasons, but most of all because it led to a deeper understanding of celestial motion. For at least 25 centuries man has been trying to reduce the complex motions of the stars, sun, moon, and planets to an orderly system. The genius of Galileo and Newton was in studying the nature of motion as it occurs on earth and then assuming that the same laws would apply to objects in the heavens beyond man's reach. Unit 2 is an account of the immense success of this idea. We shall trace the line of thought, starting with the formulation of the planetary problems by the ancient Greeks, through the work over a 100-year span of Copernicus, Tycho Brahe, Kepler, and Galileo, that provided a planetary model and several general laws for planetary motion, to Newton's magnificent synthesis of terrestrial and celestial physics in his Law of Universal Gravitation.



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Prologue

P. 4 U.S. Atomic Energy Commission.

P. 6 (left) Mt. Wilson and Palomar Observatories: (right) Professor Erwin W. Mueller, The Pennsylvania State University.

P. 7 (left) Museum of Comparative Zoology, Harvard University; (right) Brookhaven National Laboratory.

Chapter 1

P. 8 Yale University Art Gallery, Collection Societe Anonyme.

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Chapter 2

P. 36 Cabinet des Dessins, Louvre Museum.

P. 39 Vatican Museum, Rome.

P. 41 (signature) Smith Collection, Columbia University Libraries.

Pp. 43,44 Houghton Library, Harvard University.

P. 45 Courtesy of Educational Development Center, Newton, Mass.

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P. 56 (top two) Bethand, Ferdinand, Histoire de la mésure du temps, Paris, 1802; (bottom two) The Science Museum, London.

P. 63 Official U.S. Air Force photo.

Chapter 3

P. 64 A.G. Mili, © Time Inc.

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P. 80 (balance) Collection of Historical Scientific Instruments, Harvard University.

P. 81 Dr. Harold E. Edgerton, MIT.

Pp. 82, 83 National Aeronautics and Space Administration.

P. 89 U.S. Air Force.

Chapter 4

P. 93 National Aeronautics and Space Administration; Verne, Jules, De la terre à la lune, Paris, 1866.

P. 94 (top) The Boeing Corp.

P. 95 NASA.

P. 96 (skater) National Film Board of Canada; (fireworks) Stan Wayman, LIFE MAGAZINE, © Time Inc.

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Answers to End of Section Questions

Chapter 1

Q1 .5 cm/sec from 0 to 2 seconds; .33 cm/sec from 2 to 5 seconds; 2 cm/sec from 5 to 6 seconds.

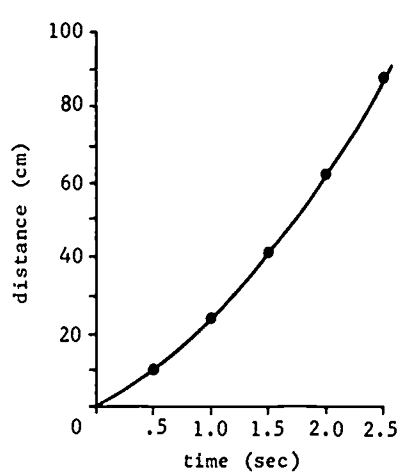
Q2 .66 cm/sec.

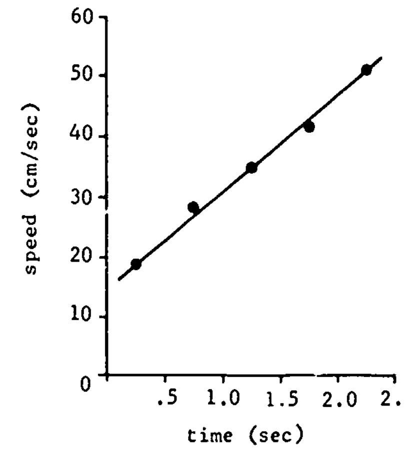
Q3 Interpolation means estimating value <u>between</u> data points; extrapolation means estimating values beyond data points.

Q4 The average speed over an interval of time Δt is $\Delta d/\Delta t$; instantaneous speed means in principle the speed at a point, and in practice is defined as the average speed for an interval so small that the average speed wouldn't change if the interval were made smaller.

Q5 The following table summarizes the data in the photo on page 28. The two accompanying graphs are based on the same data.

position	s	t	v
0	0 cm	0 sec	
			19 cm/sec
1	9.5	0.5	
			28
2	23.5	1.0	
			35
3	41	1.5	
			42
4	62	2.0	
			51
5	87.5	2.5	





Why doesn't the speed-time graph pass through the origin?

Q6 40,000 miles/hour^2 or 12 mph/sec.

Q7 -8 miles/hour^2.

Chapter 2

Q1 He could not measure v.

Q2 d = vt can only be used if v is constant. In acceleration motion v is not constant and the two equations cannot be combined.

Q3 c

Q4 c (a case can also be made for (a) or (b))

Q5 a

Chapter 3

Q1 Speed is a scalar quantity having only a magnitude while velocity is a vector quantity having both a magnitude and a direction.

Q2 a) $\Delta\vec{v}$ = 14.1 m/sec southeast
b) $\vec{a}$ = 2.8 m/sec^2 southeast

Q3 Force

Q4 According to Aristotelian physics a force is needed to maintain a motion. One possible (but slightly unbelievable) explanation would be "air currents circulate around the puck and push it along."

Q5 d

Q6 The net force acting on the puck is zero. Therefore the velocity does not change.

Q7 Galileo's "straight" lines were actually great circles about the earth. Newton's straight lines were straight.

Q8 2.5 kg

Q9 False

Q10 a) 2 m/sec^2
b) 4 N
c) Friction

Q11 c and f

Q12 e and f

Q13 No. The force "pulling the string apart" is still only 300 N.

Chapter 4

Q1 a) 2 seconds
b) 2,000 meters

Q2 a) No air resistance
b) The motion in the horizontal direction has no effect on the motion in the vertical direction.

Q3 $y = \frac{a_g}{2(v_x)^2} \times x^2$ where a_g is the acceleration due to gravity at the moon's surface.

Q4 $\frac{a_g}{2(v_x)^2}$

Q5 In cases a, b and c the results would be identical. In cases d and e the acceleration of the ball would be constant but the acceleration would be greater in case d than in case e.

Q6 Case d

Q7 a) 1.33 seconds/cycle
b) .022 minutes
c) .75 cycles/sec

Q8 3.1 m/sec^2

Q9 .77 m/sec^2

Brief Answers to Study Guide

Chapter 1

1.2 a) 6 cm/sec
b) 15 miles
c) 15 sec
d) 3 m/sec
e) 40 miles/hr
f) 40 miles/hr
g) 5.5 sec
h) 8.8 m

1.3 1.99 miles/hr

1.5 2.7×10^8 seconds

1.6 a) 1.65 m/sec
b) 3 m/sec

1.12 3.15×10^5 cm/sec

1.14 a) Approximately 25 meters
b) No

1.17 40 mph

Chapter 2

2.8 a) True
b) True
c) False
d) True (if air resistance is present)

2.15 c) 8 hours

2.16 a) 4.9 m
b) 9.8 m/sec
c) 14.7 m

2.17 a) 10.2 m/sec
b) 15.1 m
c) 2.04 sec
d) 20.4 m
e) 20 m/sec

2.18 a) 20.4 m/sec
b) 18.8 m/sec
c) 4.08 sec
d) 81.6 m
e) 0 (It is on the ground.)
f) 40 m/sec

2.19 a) 2 m/sec^2
b) 2 m/sec
c) 2 m/sec
d) 4 m
e) 2 m/sec
f) 4 sec

2.20 a) 56.8 m/sec^2
b) 710 m (approximately)
c) 189 m/sec^2 (about 19.5 g's)

2.23 a) 4.30 $welfs/surgs^2$
b) a_g = 980 cm/sec^2 or 9.8 m/sec^2. The planet Arret could be similar to the planet earth.

Chapter 3

3.3 a) Yes
b) 4.2 units 20^0 south of west

3.8 6:1

3.13 2 kg

3.14 a) a = 201 m/sec^2 v = 790 m/sec
b) The mass of the rocket decreases as propellent leaves the rocket.
c) 220 m/sec^2 (The acceleration is not uniform.)

3.16 a) 850 N
b) 735 N
c) 622 N
d) 850 N, 735 N, 622 N
e) The bathroom scale indicates a weight change.

3.17 a) 1 kg
b) 9.81 N, 9.80 N

3.18 b) 1.6×10^{-24} m/sec
c) 6.0×10^{24}:1

Chapter 4

4.2 e

4.3 a) x = 1.5 m, y = 1.25 m, $\vec{d}$ = 1.9 m at angle 40^0 below horizontal
b) $\vec{v}$ = 5.7 m/sec at angle 59^0 below horizontal

4.8 a) 10.2 meters
b) 61.2 meters

4.11 a) 0.2 seconds
b) 5 cps
c) 62.8 m/sec

4.13 a) Height above surface
b) Syncom 2
c) Lunik 3 and Luna 4
d) Luna 4
e) 24.3 hours
f) Remains almost directly above that spot
g) Midas 3
h) 9.4 m/sec^2

4.19 For the nose wheels, $a_c = 1.8 \times 10^4$ m/sec^2.

4.20 The centripetal acceleration of the scooter wheel would be twice that of the motor cycle wheel.

4.21 a) $a_c = 2.2 \times 10^{-10}$ m/sec^2
b) $F_c = 4 \times 10^{20}$ N
c) $F = 3.55 \times 10^{22}$ N
d) $v = 2.98 \times 10^4$ m/sec

Acknowledgments

Prologue

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P. 81 Newton, Sir Isaac, The Principia, Vol. I, Mott's translation revised by Florian Cajori, U. of Calif. Press, pp. 13-14.

Pp. 86-87 Ibid., pp. XIII-XV.

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P. 92 Newton, Sir Isaac, op. cit., Vol. II, p. 551.

P. 111 Childe, V. Gordon, "Rotary Motion," A History of Technology, Vol. I, Oxford University Press, p. 187.

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P. 127 Whitehead, A. N., Science and the Modern World, a Mentor Book published by The New American Library, pp. 46-47.